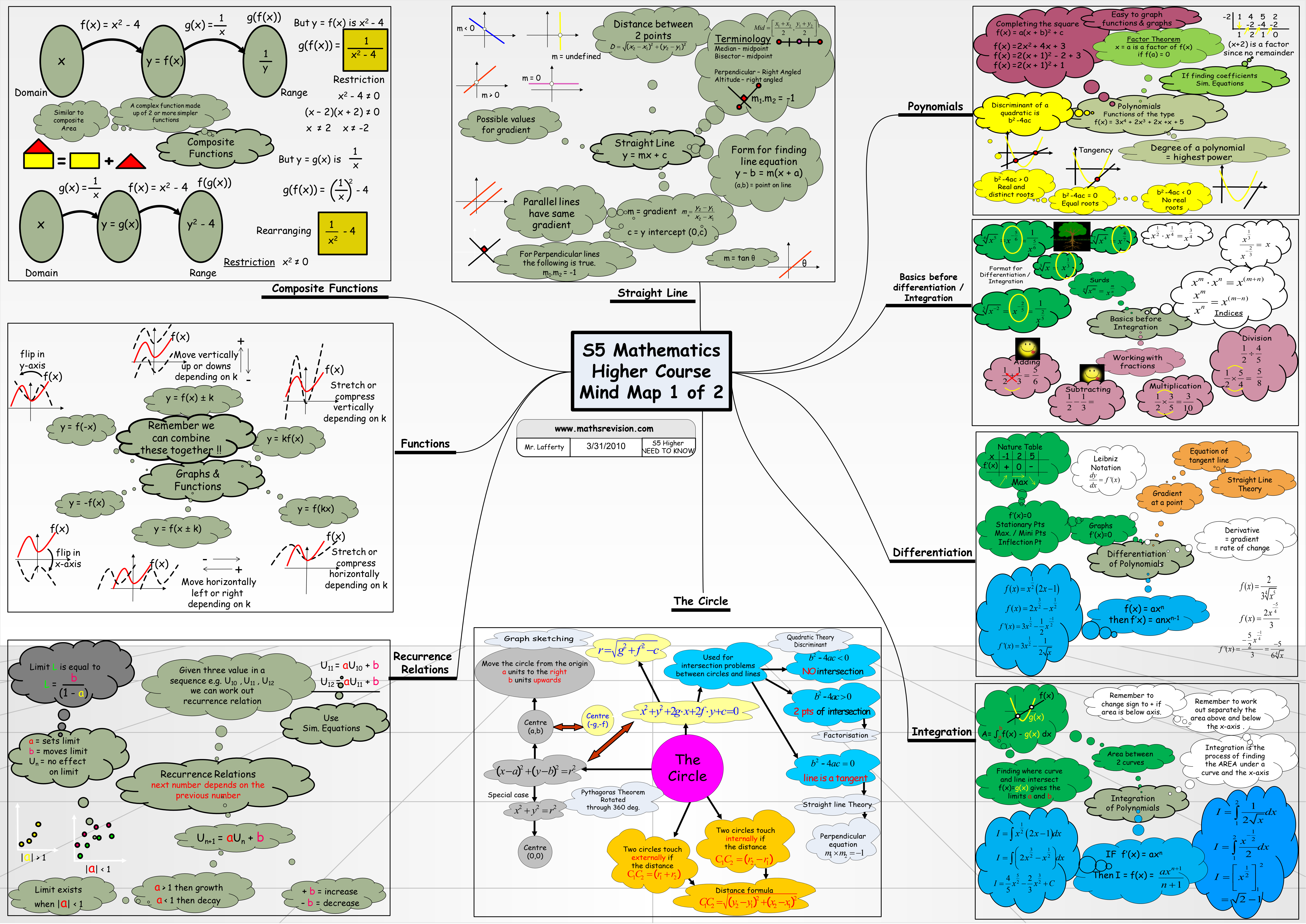

S5 Mathematics Higher Course Mind Map 1 of 2
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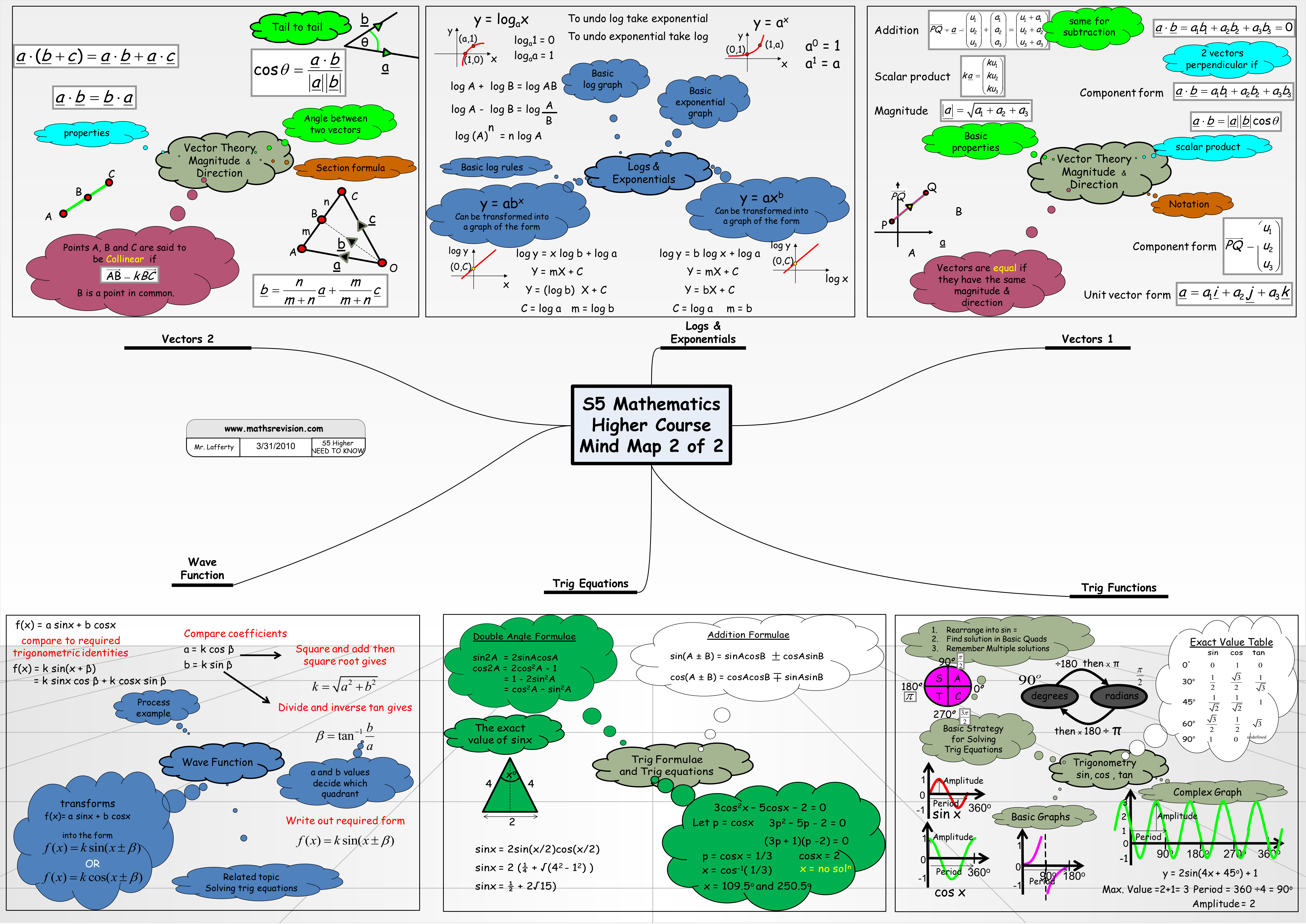
Mr. Lafferty
3/31/2010
S5 Higher NEED TO KNOW
Composite Functions
f(x) = x² - 4
g(x) = 1/x
g(f(x))
Domain
Range
But y = f(x) is x² - 4
g(f(x)) = 1/(x² - 4)
Restriction
x² - 4 ≠ 0
(x - 2)(x + 2) ≠ 0
x ≠ 2 x ≠ -2
Similar to composite Area
A complex function made up of 2 or more simpler functions
Composite Functions
But y = g(x) is 1/x
f(g(x))
g(f(x)) = (1/x)² - 4
Rearranging
1/x² - 4
Restriction x² ≠ 0
Straight Line
m < 0
m = undefined
m > 0
m = 0
Distance between 2 points
Terminology
Median - midpoint
Bisector - midpoint
Perpendicular - Right Angled
Altitude - right angled
m₁.m₂ = -1
Possible values for gradient
Straight Line y = mx + c
Form for finding line equation y - b = m(x - a)
(a,b) = point on line
Parallel lines have same gradient
m = gradient
c = y intercept (0,c)
For Perpendicular lines the following is true. m₁.m₂ = -1
m = tan θ
Functions
flip in y-axis
Move vertically up or down depending on k
Stretch or compress vertically depending on k
y = f(x) ± k
y = f(-x)
Remember we can combine these together !!
y = kf(x)
Graphs & Functions
y = -f(x)
y = f(kx)
y = f(x ± k)
flip in x-axis
Move horizontally left or right depending on k
Stretch or compress horizontally depending on k
Recurrence Relations
Limit L is equal to L = b/(1 - a)
Given three value in a sequence e.g. U₁₀ , U₁₁ , U₁₂ we can work out recurrence relation
U₁₁ = aU₁₀ + b
U₁₂ = aU₁₁ + b
Use Sim. Equations
a = sets limit
b = moves limit
Uₙ = no effect on limit
Recurrence Relations next number depends on the previous number
Uₙ₊₁ = aUₙ + b
|a| > 1
|a| < 1
Limit exists when |a| < 1
a > 1 then growth
a < 1 then decay
+ b = increase
- b = decrease
The Circle
Graph sketching
Move the circle from the origin a units to the right b units upwards
r = √(g² + f² - c)
Quadratic Theory Discriminant
Used for intersection problems between circles and lines
b² - 4ac < 0 NO intersection
b² - 4ac > 0 2 pts of intersection
Factorisation
b² - 4ac = 0 line is a tangent
Straight line Theory
Perpendicular equation m₁ × m₂ = -1
x² + y² + 2gx + 2fy + c = 0
Centre (a,b)
Centre (-g,-f)
(x - a)² + (y - b)² = r²
Special case
Pythagoras Theorem Rotated through 360 deg.
x² + y² = r²
Centre (0,0)
The Circle
Two circles touch externally if the distance C₁C₂ = (r₁ + r₂)
Two circles touch internally if the distance C₁C₂ = (r₂ - r₁)
Distance formula C₁C₂ = √((y₂ - y₁)² + (x₂ - x₁)²)
Poynomials
Completing the square f(x) = a(x + b)² + c
f(x) = 2x² + 4x + 3
f(x) = 2(x + 1)² - 2 + 3
f(x) = 2(x + 1)² + 1
Easy to graph functions & graphs
Factor Theorem x = a is a factor of f(x) if f(a) = 0
(x+2) is a factor since no remainder
If finding coefficients Sim. Equations
Discriminant of a quadratic is b² - 4ac
Polynomials Functions of the type f(x) = 3x⁴ + 2x³ + 2x + x + 5
Tangency
Degree of a polynomial = highest power
b² - 4ac > 0 Real and distinct roots
b² - 4ac = 0 Equal roots
b² - 4ac < 0 No real roots
Basics before differentiation / Integration
Format for Differentiation / Integration
Surds
Basics before Integration
Indices
Adding
Working with fractions
Division
Subtracting
Multiplication
Differentiation
Nature Table
Max
Leibniz Notation dy/dx = f′(x)
Equation of tangent line
Straight Line Theory
Gradient at a point
f(x)=0 Stationary Pts Max. / Mini Pts Inflection Pt
Graphs f′(x)=0
Derivative = gradient = rate of change
Differentiation of Polynomials
f(x) = axⁿ then f′(x) = anxⁿ⁻¹
Integration
A = ∫ f(x) - g(x) dx
Remember to change sign to + if area is below axis.
Remember to work out separately the area above and below the x-axis .
Area between 2 curves
Integration is the process of finding the AREA under a curve and the x-axis
Finding where curve and line intersect f(x)-g(x) gives the limits a and b
Integration of Polynomials
IF f′(x) = axⁿ Then I = f(x) = axⁿ⁺¹/(n + 1)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S5 Higher NEED TO KNOW
Vectors 2
Tail to tail
$\cos\theta = \frac{\underline{a}\cdot\underline{b}}{|\underline{a}||\underline{b}|}$
$\underline{a}\cdot(\underline{b}+\underline{c}) = \underline{a}\cdot\underline{b}+\underline{a}\cdot\underline{c}$
$\underline{a}\cdot\underline{b} = \underline{b}\cdot\underline{a}$
properties
Angle between two vectors
Vector Theory Magnitude & Direction
Section formula
Points A, B and C are said to be Collinear if
$\vec{AB} = k\vec{BC}$
B is a point in common.
$\underline{b} = \frac{n}{m+n}\underline{a} + \frac{m}{m+n}\underline{c}$
Logs & Exponentials
$y = \log_a x$
To undo log take exponential
To undo exponential take log
$\log_a 1 = 0$
$\log_a a = 1$
$y = a^x$
$a^0 = 1$
$a^1 = a$
Basic log graph
Basic exponential graph
log A + log B = log AB
log A - log B = log A/B
log (A)^n = n log A
Basic log rules
$y = ab^x$ Can be transformed into a graph of the form
log y = x log b + log a
Y = mX + C
Y = (log b) X + C
C = log a m = log b
$y = ax^b$ Can be transformed into a graph of the form
log y = b log x + log a
Y = mX + C
Y = bX + C
C = log a m = b
Vectors 1
Addition
same for subtraction
$\underline{a}\cdot\underline{b} = a_1b_1 + a_2b_2 + a_3b_3 = 0$
2 vectors perpendicular if
Scalar product
Component form
$\underline{a}\cdot\underline{b} = a_1b_1 + a_2b_2 + a_3b_3$
Magnitude
$|\underline{a}| = \sqrt{a_1 + a_2 + a_3}$
$\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\theta$
Basic properties
scalar product
Vector Theory Magnitude & Direction
Notation
Component form
Vectors are equal if they have the same magnitude & direction
Unit vector form
$\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$
Wave Function
f(x) = a sinx + b cosx
compare to required trigonometric identities
f(x) = k sin(x + β)
= k sinx cos β + k cosx sin β
Compare coefficients
a = k cos β
b = k sin β
Square and add then square root gives
$k = \sqrt{a^2 + b^2}$
Divide and inverse tan gives
$\beta = \tan^{-1}\frac{b}{a}$
Process example
Wave Function
a and b values decide which quadrant
transforms f(x)= a sinx + b cosx into the form
$f(x) = k\sin(x \pm \beta)$ OR $f(x) = k\cos(x \pm \beta)$
Write out required form
$f(x) = k\sin(x \pm \beta)$
Related topic Solving trig equations
Trig Equations
Double Angle Formulae
sin2A = 2sinAcosA
cos2A = 2cos²A - 1
= 1 - 2sin²A
= cos²A - sin²A
Addition Formulae
sin(A ± B) = sinAcosB ± cosAsinB
cos(A ± B) = cosAcosB ∓ sinAsinB
The exact value of sinx
sinx = 2sin(x/2)cos(x/2)
sinx = 2 (¼ + √(4² - 1²))
sinx = ½ + 2√15)
Trig Formulae and Trig equations
3cos²x - 5cosx - 2 = 0
Let p = cosx 3p² - 5p - 2 = 0
(3p + 1)(p -2) = 0
p = cosx = 1/3 cosx = 2
x = cos⁻¹(1/3) x = no solⁿ
x = 109.5° and 250.5°
Trig Functions
1. Rearrange into sin =
2. Find solution in Basic Quads
3. Remember Multiple solutions
Basic Strategy for Solving Trig Equations
÷180 then × π
then × 180 ÷ π
degrees
radians
Exact Value Table
Trigonometry sin, cos , tan
Complex Graph
Basic Graphs
sin x
cos x
Amplitude
Period
y = 2sin(4x + 45°) + 1
Max. Value =2+1= 3 Period = 360 ÷4 = 90°
Amplitude = 2