Integration Past Papers Unit 2 Outcome 2

Multiple Choice Questions

Each correct answer in this section is worth two marks.

1. Evaluate $\int_1^4 x^{-1/2}\,dx$.

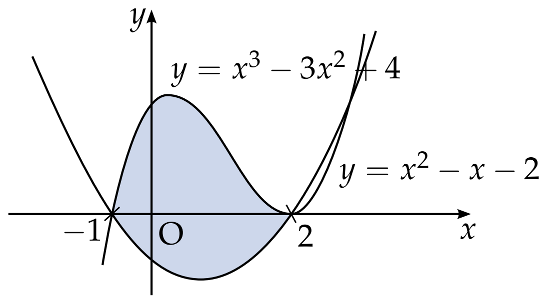
   A. -2

   B. $-\frac{7}{16}$

   C. $\frac{1}{2}$

   D. 2

2. The diagram shows the area bounded by the curves $y = x^3 - 3x^2 + 4$ and $y = x^2 - x - 2$ between $x = -1$ and $x = 2$.



   Represent the shaded area as an integral.

   A. $\int_{-1}^{2} \left(x^3 - 4x^2 + x + 6\right) dx$

   B. $\int_{-1}^{2} \left(-x^3 + 4x^2 - x - 6\right) dx$

   C. $\int_{-1}^{2} \left(x^3 - 4x^2 - x + 2\right) dx$

   D. $\int_{-1}^{2} \left(x^3 - 2x^2 - x + 2\right) dx$

[END OF MULTIPLE CHOICE QUESTIONS]

Written Questions

[SQA] 3. Find $\int \left(2x^2 + 3\right) dx$. **3**

[SQA] 4. Find $\int \left(3x^3 + 4x\right) dx$. **3**

[SQA] 5. Evaluate $\int_1^2 \left(x^2 + \frac{1}{x}\right)^2 dx$. **5**

[SQA] 6. Find $\int \frac{x^2 - 5}{x\sqrt{x}}\, dx$. **4**

[SQA] 7. Find $\int \frac{(x^2 - 2)(x^2 + 2)}{x^2}\, dx$, $x \neq 0$. **4**

[SQA] 8. Find the value of $\int_1^2 \frac{u^2 + 2}{2u^2}\, du$. **5**

[SQA] 9.

(*a*) Find the value of $\int_1^2 \left(4 - x^2\right) dx$. **3**

(*b*) Sketch a graph and shade the area represented by the integral in (*a*). **2**

[SQA] 10. Evaluate $\int_1^9 \frac{x + 1}{\sqrt{x}}\, dx$. **5**

[SQA] 11. Find the value of $\int_1^4 \sqrt{x}\, dx$. **4**

[SQA] 12. Find $\int \frac{1}{(7 - 3x)^2}\, dx$. **2**

[SQA] 13. Evaluate $\int_{-3}^0 \left(2x + 3\right)^2 dx$. **4**

[SQA] 14. Evaluate $\int_1^2 \left(3x^2 + 4\right) dx$ and draw a sketch to illustrate the area represented by this integral. **5**

[SQA] 15.

(*a*) Find the coordinates of the points of intersection of the curves with equations $y = 2x^2$ and $y = 4 - 2x^2$. **2**

(*b*) Find the area completely enclosed between these two curves. **3**

[SQA] 16. For all points on the curve $y = f(x)$, $f'(x) = 1 - 2x$.

If the curve passes through the point $(2, 1)$, find the equation of the curve. **4**

[SQA] 17. A curve for which $\dfrac{dy}{dx} = 3x^2 + 1$ passes through the point $(-1, 2)$.

Express y in terms of x. **4**

[SQA] 18. A curve for which $\dfrac{dy}{dx} = 6x^2 - 2x$ passes through the point $(-1, 2)$.

Express y in terms of x. **3**

[SQA] 19. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

[SQA] 20. A curve with equation $y = f(x)$ passes through the point $(2, -1)$ and is such that $f'(x) = 4x^3 - 1$.

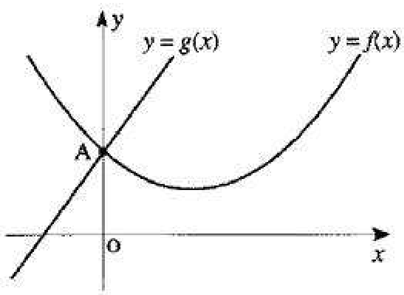
Express $f(x)$ in terms of x. **5**

[SQA] 21. The graph of $y = g(x)$ passes through the point $(1, 2)$.

If $\dfrac{dy}{dx} = x^3 + \dfrac{1}{x^2} - \dfrac{1}{4}$, express y in terms of x. **4**

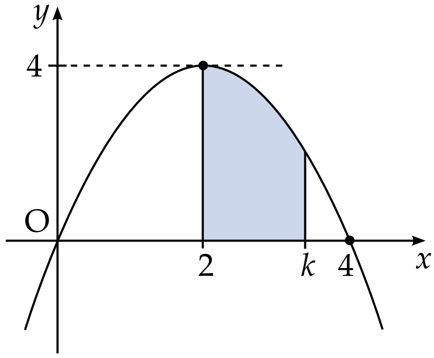
[SQA] 22. The graphs of $y = f(x)$ and $y = g(x)$ intersect at the point A on the y-axis, as shown in the diagram.

If $g(x) = 3x + 4$
and $f'(x) = 2x - 3$, find $f(x)$. **4**



[SQA] 23. The parabola shown crosses the x-axis at $(0, 0)$ and $(4, 0)$, and has a maximum at $(2, 4)$.

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.

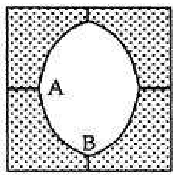


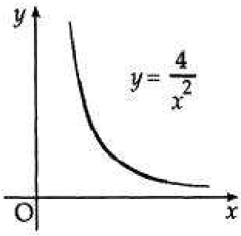
(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

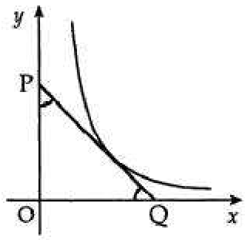
$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$. **3**

[SQA] 24. The makers of "OLO", the square mint with the not-so-round hole, commissioned an advertising agency to prepare an illustration to the specification described in (i) to (iii) below. The finished illustration will look like the diagram on the right.

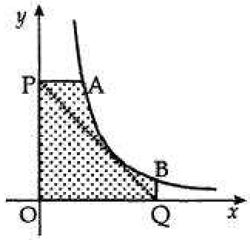




(i) The curve AB in the finished illustration is part of the curve with equation $y = \dfrac{4}{x^2}$.



(ii) A tangent to this curve, making equal angles with both axes, is to be drawn as shown (line PQ)



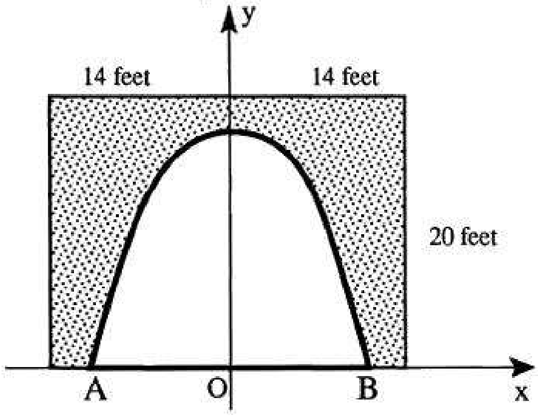
(iii) Straight lines perpendicular to the axes are to be drawn from P and Q as shown. The shaded part forms $\frac{1}{4}$ of the finished illustration.

(*a*) State the gradient of PQ and hence find the point of contact of the tangent PQ with the curve. **(5)**

(*b*) Find the equation of PQ and the coordinates of A and B. **(4)**

(*c*) Calculate the shaded area of the finished illustration. **(6)**

[SQA] 25. The concrete on the 20 feet by 28 feet rectangular facing of the entrance to an underground cavern is to be repainted.



Coordinate axes are chosen as shown in the diagram with a scale of 1 unit equal to 1 foot.The roof is in the form of a parabola with equation $y = 18 - \frac{1}{8}x^2$.

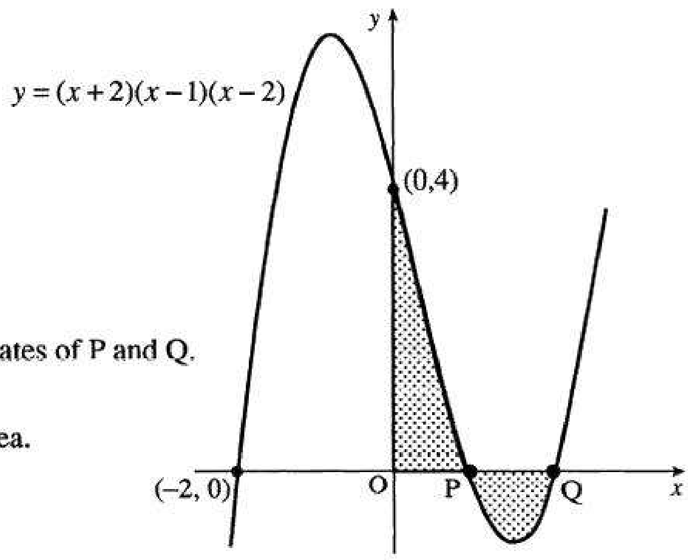
(*a*) Find the coordinates of the points A and B. **(2)**

(*b*) Calculate the total cost of repainting the facing at £3 per square foot. **(4)**

[SQA] 26. The diagram shows a sketch of the graph of $y = (x+2)(x-1)(x-2)$. The graph cuts the axes at (–2, 0), (0, 4) and the points P and Q.

(*a*) Write down the coordinates of P and Q. **(2)**

(*b*) Find the total shaded area. **(7)**



[SQA] 27. A parabola passes through the points (0, 0), (6, 0) and (3, 9) as shown in Diagram 1.

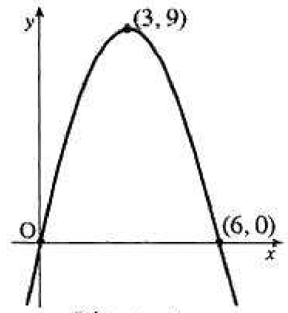


Diagram 1

(*a*) The parabola has equation of the form $y = ax(b - x)$. Determine the values of a and b. **(2)**

(*b*) Find the area enclosed by the parabola and the x-axis. **(4)**

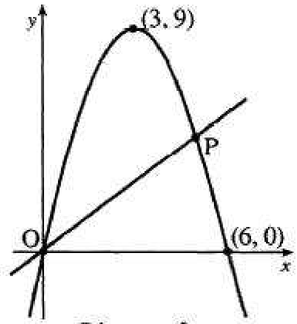


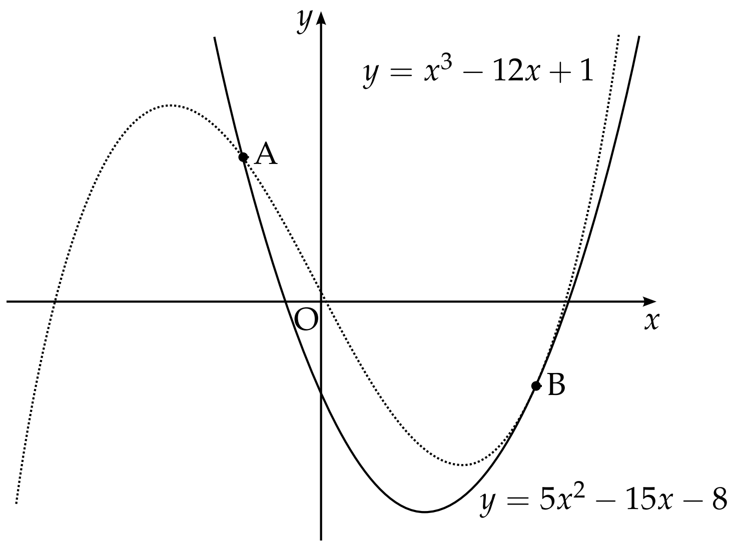
Diagram 2

(*c*) (i) Diagram 2 shows the parabola from (*a*) and the straight line with equation $y = x$. Find the coordinates of P, the point of intersection of the parabola and the line.

(ii) Calculate the area enclosed between the parabola and the line. **(5)**

[SQA] 28. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(a) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(b) The point A is $(-1, 12)$ and B is $(3, -8)$.

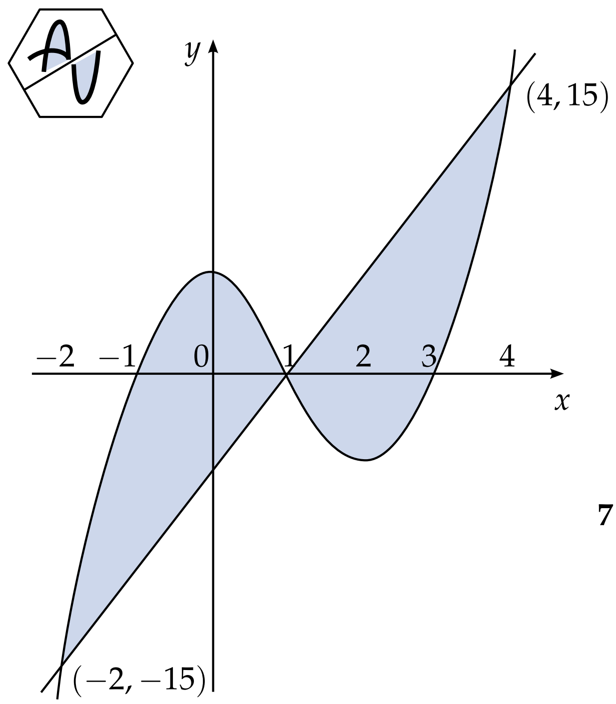
Find the area enclosed between the two curves. **5**

[SQA] 29. A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

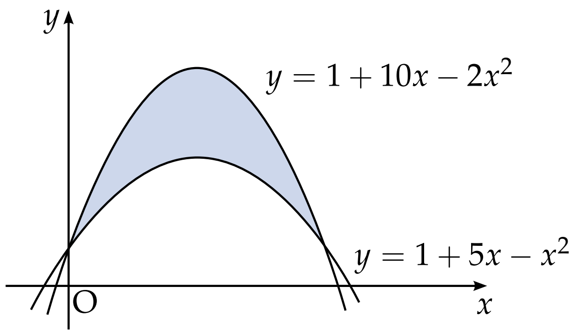
A mathematical representation of the final logo is shown in the coordinate diagram.

The curve has equation $y = (x+1)(x-1)(x-3)$ and the straight line has equation $y = 5x - 5$. The point $(1, 0)$ is the centre of half-turn symmetry.

Calculate the total shaded area. **7**



[SQA] 30. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. **6**

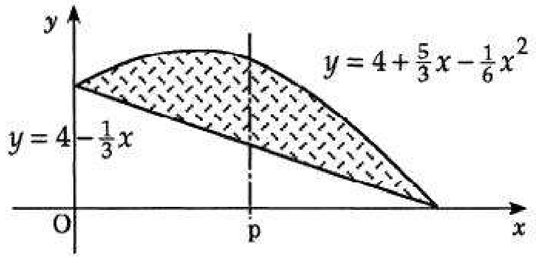


[SQA] 31. When building a road beside a vertical rockface, engineers often use wire mesh to cover the rockface. This helps to prevent rocks and debris from falling onto the road. The shaded region of the diagram below represents a part of such a rockface.

This shaded region is bounded by a parabola and a straight line.

The equation of the parabola is $y = 4 + \frac{5}{3}x - \frac{1}{6}x^2$ and the equation of the line is $y = 4 - \frac{1}{3}x$.

(*a*) Find algebraically the area of wire mesh required for this part of the rockface. **(5)**



(*b*) To help secure the wire mesh, weights are attached to the mesh along the line $x = p$ so that the area of mesh is bisected.

By using your answer to part (*a*), or otherwise, show that

$$p^3 - 18p^2 + 432 = 0.$$ **(3)**

(*c*) (i) Verify that $p = 6$ is a solution of this equation.

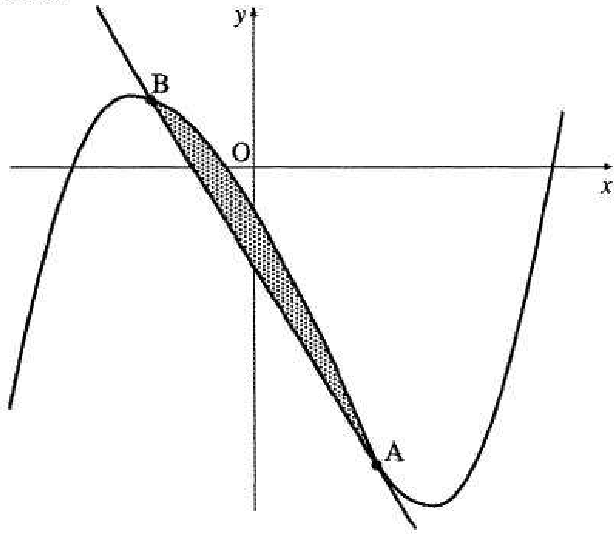
(ii) Find algebraically the other two solutions of this equation.

(iii) Explain why $p = 6$ is the only valid solution to this problem. **(5)**

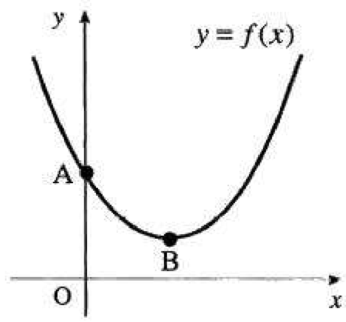
[SQA] 32. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A$(1, -8)$.

(*a*) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(*b*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**

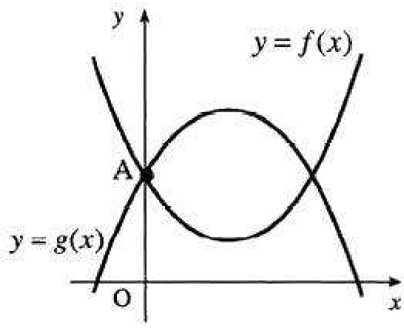


[SQA] 33. The first diagram shows a sketch of part of the graph of $y = f(x)$ where $f(x) = (x-2)^2 + 1$. The graph cuts the y-axis at A and has a minimum turning point at B.



(*a*) Write down the coordinates of A and B. **(3)**

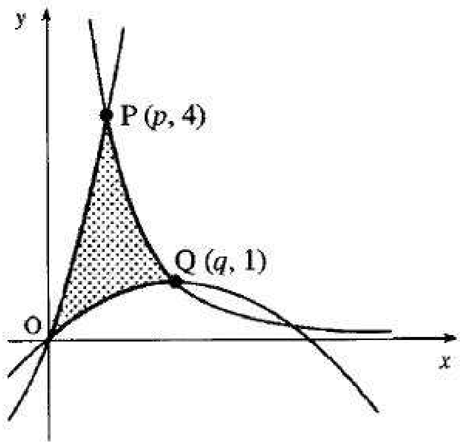
(*b*) The second diagram shows the graphs of $y = f(x)$ and $y = g(x)$ where $g(x) = 5 + 4x - x^2$.
Find the area enclosed by the two curves. **(5)**



(*c*) $g(x)$ can be written in the form $m + n \times f(x)$ where m and n are constants.
Write down the values of m and n. **(2)**

[SQA] 34. The origin, O, and the points P and Q are the vertices of a curved 'triangle' which is shaded in the diagram. The sides lie on curves with equations

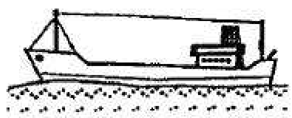
$y = x(x+3)$, $y = x - \frac{1}{4}x^2$ and $y = \frac{4}{x^2}$.



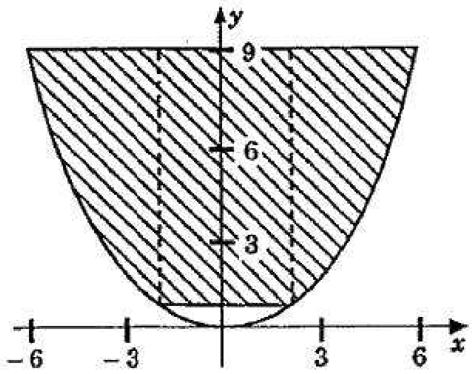
(*a*) P and Q have coordinates $(p, 4)$ and $(q, 1)$. Find the values of p and q. **2**

(*b*) Calculate the shaded area . **7**

[SQA] 35. The cargo space of a small bulk carrier is 60m long.



The shaded part of the diagram represents the uniform cross-section of this space. It is shaped like the parabola with equation $y = \frac{1}{4}x^2$, $-6 \leq x \leq 6$, between the lines $y = 1$ and $y = 9$. Find the area of this cross-section and hence find the volume of cargo that this ship can carry.



(9)

[SQA] 36. Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ and $g(x) = x^2$.

(*a*) Find formulae for

(i) $f\big(g(x)\big)$

(ii) $g\big(f(x)\big)$. **4**

(*b*) The function h is defined by $h(x) = f\big(g(x)\big) + g\big(f(x)\big)$.

Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h. **3**

(*c*) Find the area enclosed between this graph and the x-axis. **4**

[SQA] 37. A function f is defined by the formula $f(x) = 4x^2(x-3)$ where $x \in \mathbb{R}$.

(*a*) Write down the coordinates of the points where the curve with equation $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the stationary points of $y = f(x)$ and determine the nature of each. **6**

(*c*) Sketch the curve $y = f(x)$. **2**

(*d*) Find the area completely enclosed by the curve $y = f(x)$ and the x-axis. **4**

[END OF WRITTEN QUESTIONS]