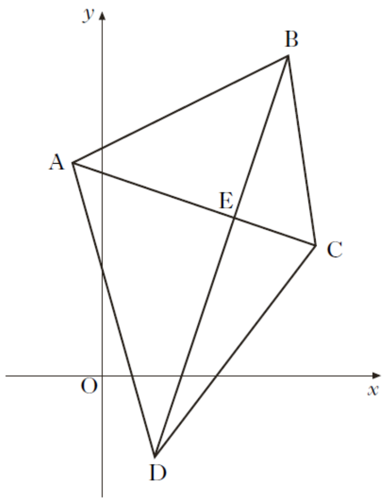
MILLBURN ACADEMY

MATHS DEPARTMENT

HIGHER MATHS HOMEWORK BOOKLET

The Straight Line

1) A quadrilateral has vertices A(–1, 8), B(7, 12), C(8, 5) and D(2, –3) as shown in the diagram.



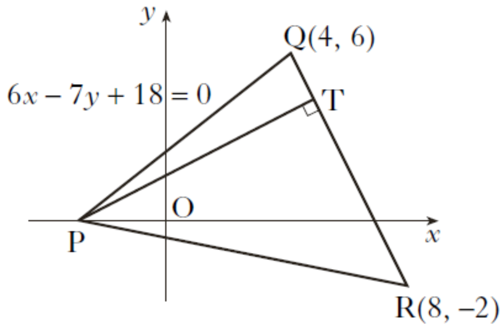
(*a*) Find the equation of diagonal BD. (2)

(*b*) The equation of diagonal AC is $x + 3y = 23$. Find the coordinates of E, the point of intersection of the diagonals. (3)

(*c*) (i) Find the equation of the perpendicular bisector of AB.

(ii) Show that this line passes through E. (5)

2) Triangle PQR has vertex P on the *x*-axis, as shown in the diagram.
Q and R are the points (4, 6) and (8, –2) respectively.
The equation of PQ is $6x - 7y + 18 = 0$.

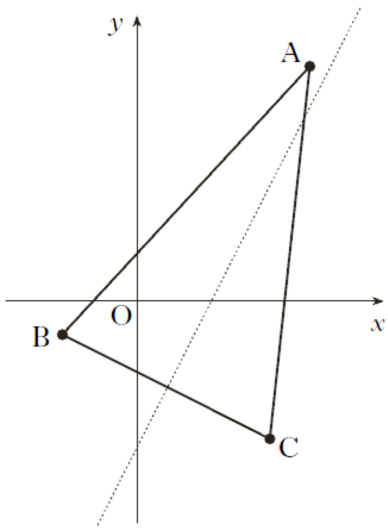


(*a*) State the coordinates of P. (1)

(*b*) Find the equation of the altitude of the triangle from P. (3)

(*c*) The altitude from P meets the line QR at T. Find the coordinates of T. (4)

3) The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.
The broken line represents the perpendicular bisector of BC.



(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. (4)

(*b*) Find the equation of the median from C. (3)

(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. (3)

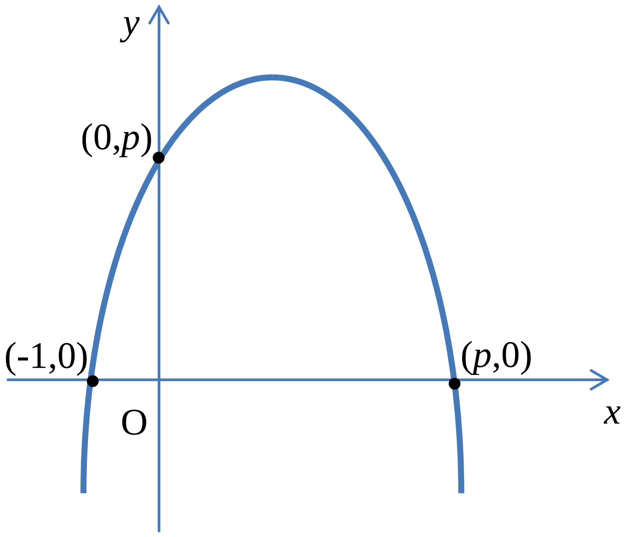
(28)

Quadratics

1) For what values of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? (3)

2) a) Given $f(x) = x^2 + 2x - 8$, express $f(x)$ in the form $(x + a)^2 - b$. (2)
   b) State the minimum value of the function $f(x)$. (1)

3) The diagram shows a sketch of a parabola passing through (-1,0), (o,p) and (p,0).



   a) Show that the equation of the parabola is $y = p + (p - 1)x - x^2$. (3)

   b) For what value of p will the line $y = x + p$ be a tangent to this curve? (3)

4) Show that the equation $(1 - 2k)\,x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. (5)

5) a) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. (2)
   b) Hence or otherwise sketch the graph of $y = f(x)$. (2)

6) Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. (5)

7) Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p. (4)

(30)

Polynomials

1) (a) Given $(x + 2)$ is a factor of $2x^3 + x^2 + kx + 2$, find the value of k. (3)

(b) Hence solve the equation $2x^3 + x^2 + kx + 2 = 0$, when k takes this value. (2)

2) Given $(x – 2)$ and $(x + 3)$ are factors of $f(x) = 3x^3 + 2x^2 + cx + d$

Find the values of c and d. (5)

3) The graph of $f(x) = 2x^3 – 5x^2 – 3x + 1$ has a root between 0 and 1.

Find the value of this root to one decimal place. (3)

4) $f(x) = 6x^3 – 5x^2 – 17x + 6$

(a) Show $(x – 2)$ is a factor of $f(x)$. (2)

(b) Express $f(x)$ in its fully factorised form. (2)

5) $f(x) = x^3 – x^2 – 5x – 3$

(a) (i) Show $(x + 1)$ is a factor of $f(x)$

(ii) Hence or otherwise factorise $f(x)$ fully. (5)

(b) One of the turning points of the graph lies on the x-axis.

Write down the coordinates of this turning point. (1)

6) $f(x) = 2x^3 – 7x^2 + 9$

Show $(x – 3)$ is a factor of $f(x)$ and factorise $f(x)$ fully. (5)

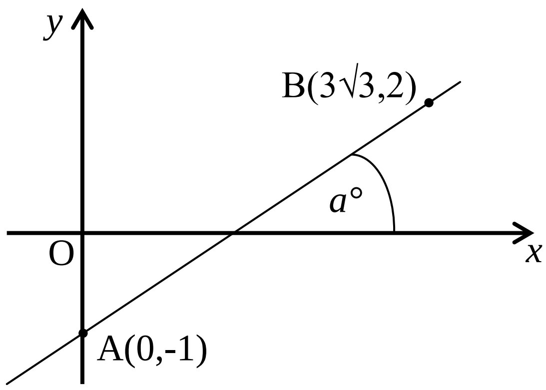
7) (a) Show $x = -1$ is a solution of $x^3 + px^2 + px + 1 = 0$ (1)

(b) Hence find the range of values of p for the equation to have real roots. (7)

(36)

Mixed Exercise 1

1) Find the size of the angle $a°$ that the line joining the points A(0,-1) and B($3\sqrt{3}$,2) makes with the positive direction of the x-axis. (3)



2) a) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$ (2)

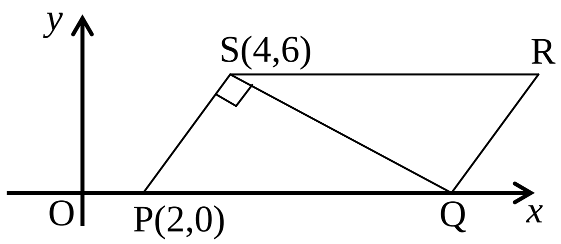
b) On the same diagram sketch:

i) the graph of $y = f(x)$

ii) the graph of $y = 10 - f(x)$ (4)

c) Find the range of values of x for which $10 - f(x)$ is positive. (1)

3) PQRS is a parallelogram. P is the point (2,0), S is (4,6) and Q lies on the x-axis, as shown.
The diagonal QS is perpendicular to the side PS.



a) Show that the equation of QS is x + 3y = 22. (4)

b) Hence find the coordinates of Q and R. (2)

4) For what value of k does the equation $x^2 - 5x + (k + 6) = 0$ have equal roots? (3)

(19)

Composite Functions

1) Given $f(x) = 2x - 1$ & $g(x) = 3 - 2x$

(a) Find an expression for $k(x) = f(g(x))$ (2)

(b) Find an expression for $h(k(x))$ given $h(x) = ¼(5 - x)$ (2)

(c) What can you say about $h(x)$ and $k(x)$ and why? (1)

2) (a) (i) Find an expression for $f(x + 1)$ given $f(x) = 4x^2 - 3x + 5$ (2)

(ii) Find an expression for $f(x - 1)$ given $f(x) = 4x^2 - 3x + 5$ (2)

(iii) *Show that* $\frac{1}{2}[f(x + 1) - f(x - 1)]$ simplifies to $8x - 3$ (1)

(b) (i) Find an expression for $g(x + 1)$ given $g(x) = 2x^2 + 7x - 8$ (2)

(ii) Find an expression for $g(x - 1)$ given $g(x) = 2x^2 + 7x - 8$ (2)

(iii) *Show that* $\frac{1}{2}[g(x + 1) - g(x - 1)]$ simplifies to $4x + 7$ (1)

(c) Hence if $h(x) = 3x^2 + 5x - 1$, find an expression $\frac{1}{2}[h(x + 1) - h(x - 1)]$ (2)

3) Given $f(x) = 2x$ and $g(x) = \sin(x) + \cos(x)$ find an expression for

(a) $f(g(x))$ (b) $g(f(x))$ (1,1)

4) Find an expression for $f(f(x))$ given $f(x) = \frac{x}{1 - x}$ (3)

5) Find $f(g(x))$ given $f(x) = \frac{1}{x + 2}$ & $g(x) = \frac{1}{x} - 2$ (3)

6) Given $f(x) = \frac{1}{x^2 - 4}$ & $g(x) = 2x + 1$

(a) Find $g(f(x))$ (3)

(b) State a suitable domain. (2)

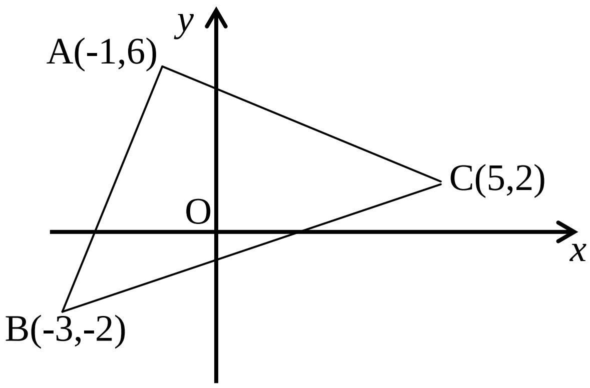
(30)

Mixed Exercise 2

1) Find the equation of the straight line which is parallel to the line with equation $2x + 3y = 5$ and which passes through the point (2,-1). (3)

2) Show that the line with equation y = 2x + 1 does not intersect the parabola with equation $y = x^2 + 3x + 4$. (5)

3) Triangle ABC has vertices A(-1,6), B(-3,-2) and C(5,2).



Find:

a) the equation of the line p, the median from C of the triangle ABC. (3)

b) the equation of the line q, the perpendicular bisector of BC. (4)

c) the coordinates of the point of intersection of the lines p and q. (1)

4) Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

a) Find an expression for $h(x)$ where $h(x) = f(g(x))$. (2)

b) Write down any restriction on the domain of h. (1)

(19)

Differentiation 1

1) Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. (5)

2) A curve has equation $y = x - \frac{16}{\sqrt{x}}$, $x > 0$.
Find the equation of the tangent at the point where $x = 4$. (6)

3) The point P(x,y) lies on the curve with equation $y = 6x^2 - x^3$.
a) Find the value of x for which the gradient of the tangent at P is 12. (5)
b) Hence find the equation of the tangent at P. (2)

4) Given that $y = \sqrt{3x^2 + 2,}$ find $\frac{dy}{dx}$. (3)

5) Given that $y = 3\sin(x) + \cos(2x)$, find $\frac{dy}{dx}$. (3)

6) Differentiate $(1 + 2\sin x)^4$ with respect to x. (2)

(26)

Mixed Exercise 3

1) The point A has coordinates (7,4). The straight lines with equations $x + 3y + 1 = 0$ and $2x + 5y = 0$ intersect at B.

a) Find the gradient of AB. (3)

b) Hence show that AB is perpendicular to only one of these two lines. (5)

2) Given that $f(x) = (5x - 4)^{1/2}$, evaluate $f'(4)$. (3)

3) $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.

a) Find $p(x)$ where $p(x) = f(g(x))$. (2)

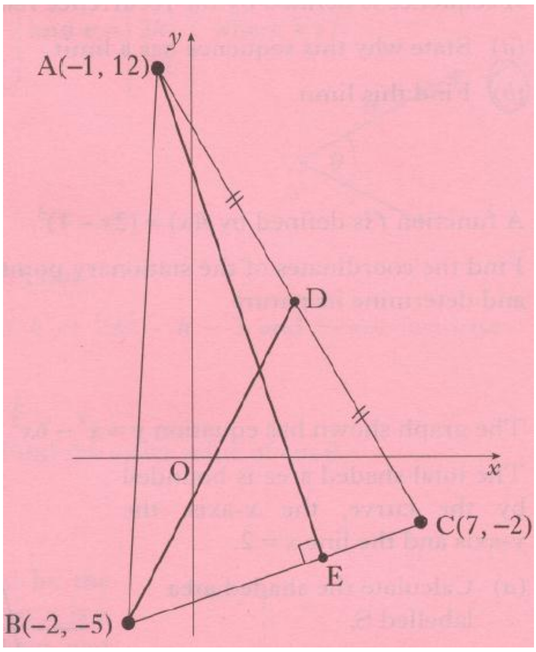
b) If $q(x) = \frac{3}{3-x}$, $x \neq 3$, find $p(q(x))$ in its simplest form. (3)

4) Triangle ABC has vertices A(-1,12), B(-2,-5) and C(7,-2).

a) Find the equation of the median BD. (3)

b) Find the equation of the altitude AE. (3)

c) Find the coordinates of the point of intersection of BD and AE. (3)



(25)

Trigonometry - Radians and Equations

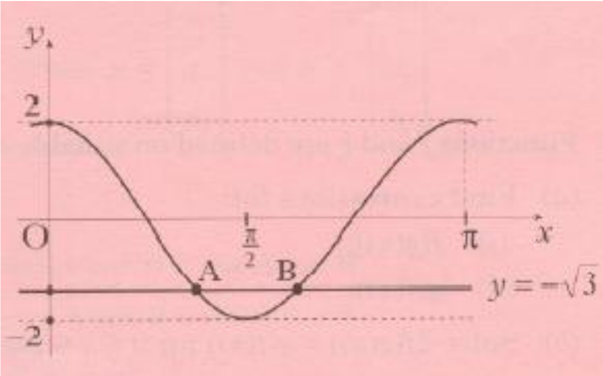
Questions 1, 3, 4 & 5 should be done WITHOUT a calculator.

1) What is the exact value of $\sin(\frac{\pi}{3})$ - $\cos\left(\frac{5\pi}{4}\right)$? (3)

2) What is the solution of the equation $4\sin x - \sqrt{7} = 0$ where $\frac{\pi}{2} \leq x \leq \pi$? (3)

3) Functions f and g are defined on suitable domains by $f(x) = \sin x$ and $g(x) = 2x$.
Find expressions for
a) $f(g(x))$ (1)
b) $g(f(x))$ (1)

4) The diagram shows the graph of a cosine function from 0 to π.



a) State the equation of the graph. (2)

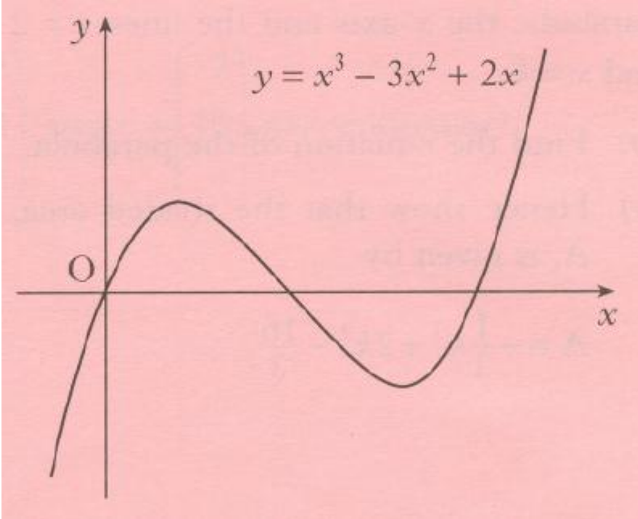
a) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B. Find the coordinates of B. (4)

5) Find all the values of x in the interval $0 \leq x \leq 2\pi$ for which $\tan^2 x = 3$. (4)

(18)

Mixed Exercise 4

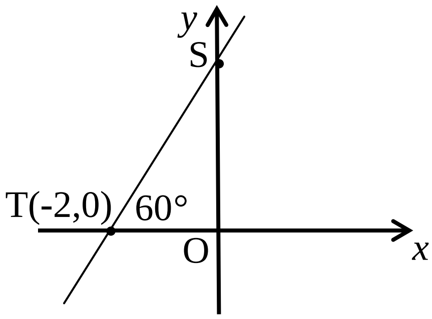
1) The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



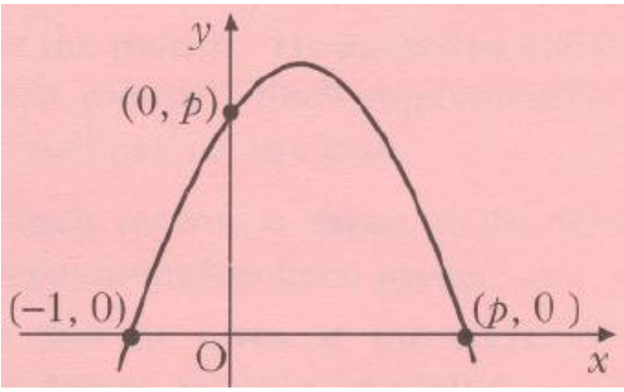
a) Find the equation of the tangent to this curve at the point where $x = 1$. (5)

b) The tangent at the point (2,0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. (5)

2) Find the equation of the line ST, where T is the point (-2,0) and angle STO is 60°. (3)



3) The diagram shows a sketch of a parabola passing through (-1,0), (0,p) and (p,0).



a) Show that the equation of the parabola is $y = p + (p-1)x - x^2$. (3)

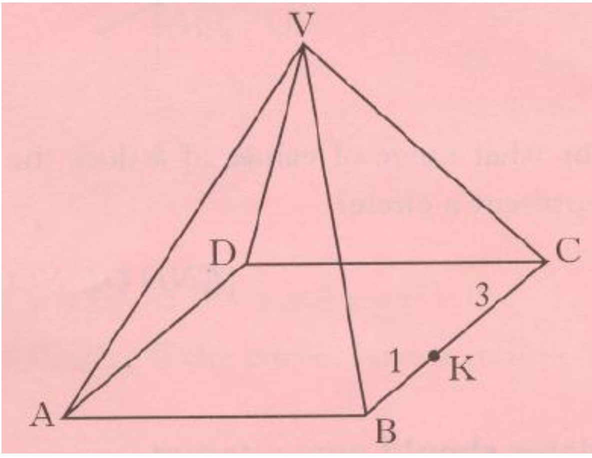
b) For what value of p will the line $y = x + p$ be a tangent to this curve? (3)

(19)

Vectors

1) VABCD is a pyramid with a rectangular base ABCD.
Relative to some appropriate axes,

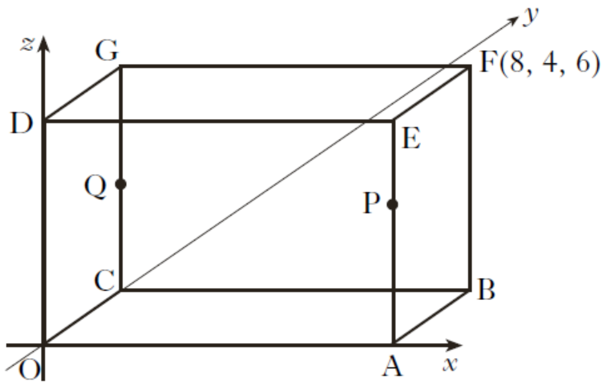
$\overrightarrow{VA}$ represents -7$\boldsymbol{i}$ – 13$\boldsymbol{j}$ – 11$\boldsymbol{k}$
$\overrightarrow{AB}$ represents 6$\boldsymbol{i}$ + 6$\boldsymbol{j}$ – 6$\boldsymbol{k}$
$\overrightarrow{AD}$ represents 8$\boldsymbol{i}$ – 4$\boldsymbol{j}$ + 4$\boldsymbol{k}$



K divides BC in the ratio 1:3.
Find $\overrightarrow{VK}$ in component form. (3)

2) Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u}$ = 3$\boldsymbol{i}$ + 2$\boldsymbol{j}$ and $\boldsymbol{v}$ = 2$\boldsymbol{i}$ – 3$\boldsymbol{j}$ + 4$\boldsymbol{k}$.
Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. (2)

3) The diagram shows a cuboid OABC, DEFG.
F is the point (8, 4, 6).
P divides AE in the ratio 2:1.
Q is the midpoint of CG.



(*a*) State the coordinates of P and Q. (2)
(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$ (2)
(*c*) Find the size of angle QPA. (5)

4) D, E and F have coordinates (10, –8, –15), (1, –2, –3) and (–2, 0, 1) respectively.
(*a*) (i) Show that D, E and F are collinear.
(ii) Find the ratio in which E divides DF. (4)
(*b*) G has coordinates (k, 1, 0).
Given that DE is perpendicular to GE, find the value of k. (4)

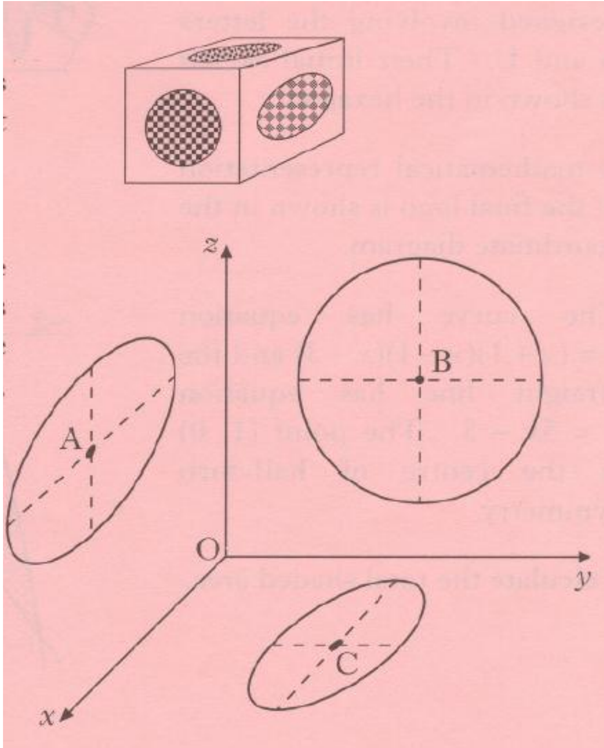
(22)

Mixed Exercise 5

1) A box in the shape of a cuboid is designed with circles of different sizes on each face.

The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are A(6,0,7), B(0,5,6) and C(4,5,0).

Find the size of angle ABC. (7)



2) Functions $f(x) = 3x - 1$ and $g(x) = x^2 + 7$ are defined on the set of real numbers.

a) Find $h(x)$ where $h(x) = g(f(x))$. (2)

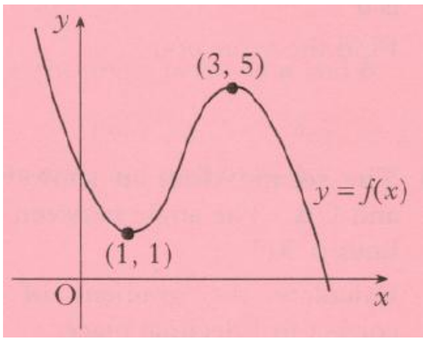
b) i) Write down the coordinates of the minimum turning point of $y = h(x)$.
ii) Hence state the range of the function h. (2)

3) If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'(\frac{\pi}{6})$. (4)

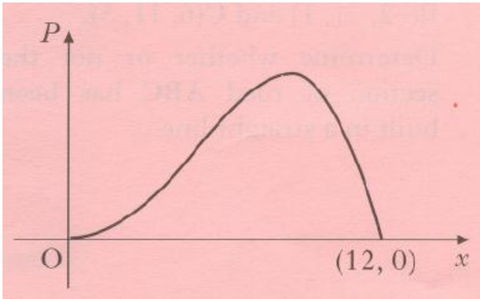
(15)

Differentiation 2

1) The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1,1) and (3,5). Sketch the graph of $y = f'(x)$. (3)



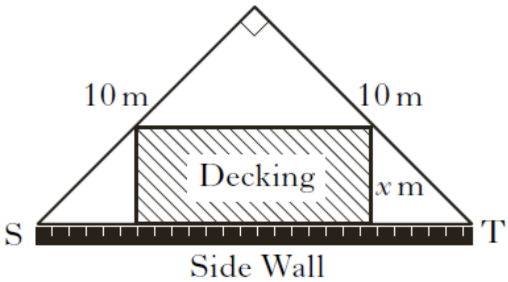
2) A company spends x thousand pounds a year on advertising and this results in a profit of P thousand pounds.
A mathematical model, illustrated in the diagram, suggests that P and x are related by $P = 12x^3 - x^4$ for $0 \le x \le 12$.
Find the value of x which gives the maximum profit. (5)



3) A function is defined by $f(x) = (2x - 1)^5$.
Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. (7)

4) A function f is defined by the formula $f(x) = 3x - x^3$.
(a) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. (2)
(b) Find the coordinates of the stationary points of the function and determine their nature. (7)
(c) Sketch the graph of $y = f(x)$. (1)

5) A householder has a garden in the shape of a right-angled isosceles triangle.
It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(a) (i) Find the exact value of ST.
(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = (10\sqrt{2})\,x - 2x^2$$ (3)

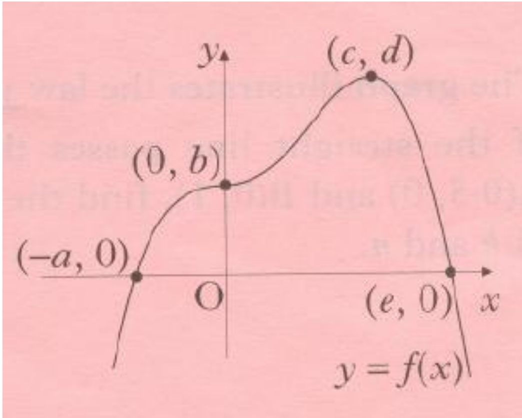
(b) Find the dimensions of the decking which maximises its area. (5)

(33)

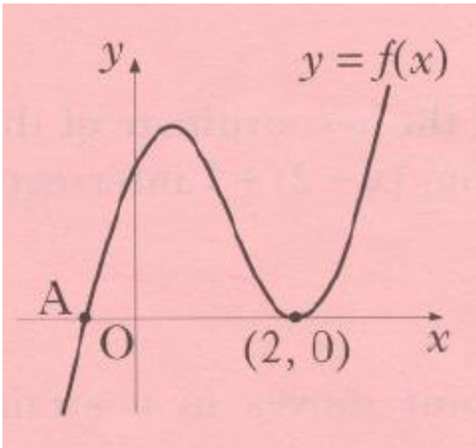
Mixed Exercise 6

1) For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? (2)

2) The graph of a function f intersects the x-axis at $(-a,0)$ and (e,o) as shown. There is a point of inflexion at $(0,b)$ and a maximum turning point (c,d). Sketch the graph of the derived function f ‘. (3)



3) The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



a) Find the x-coordinate of the maximum turning point. (5)

b) Factorise $2x^3 - 7x^2 + 4x + 4$. (3)

c) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. (2)

(15)

Trig - Addition & Double Angle Formulae

Questions 1, 3, 4 & 5 should be done WITHOUT a calculator.

1) a) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. (3)

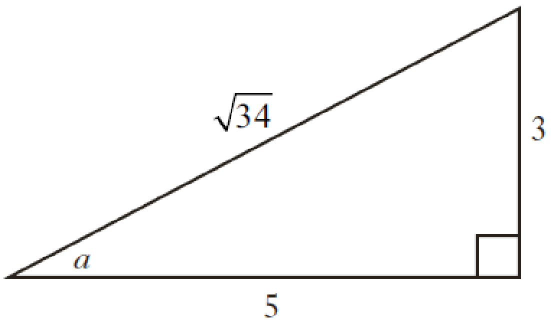
b) Show that sin(A + B) + sin(A – B) = 2sin A cos B. (2)

c) i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. (4)

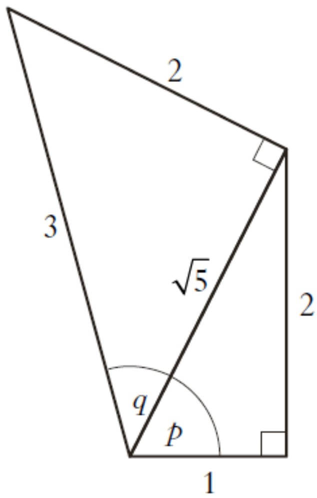
2) Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 < x < 2\pi$. (5)

3) A right-angled triangle has sides and angles as shown in the diagram.
What is the exact value of sin 2*a*? (2)



4) If the exact value of cos *x* is $\frac{1}{\sqrt{5}}$, find the exact value of cos 2*x*. (3)

5) The diagram shows two right-angled triangles with sides and angles given.
What is the exact value of sin (*p* + *q*)? (3)

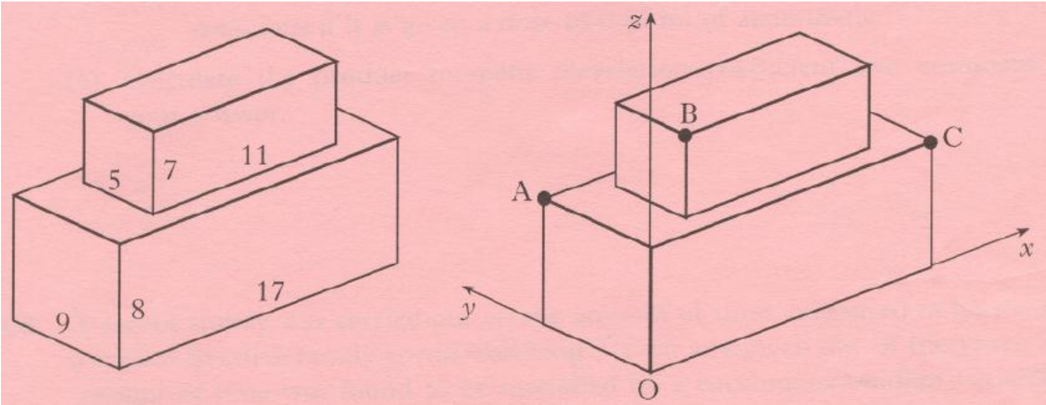


(22)

Mixed Exercise 7

1) Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \le x \le 360$. (5)

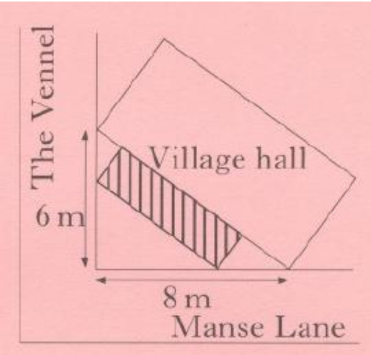
2) A cuboid measuring 11cm by 5cm by 7cm is placed centrally on top of another cuboid measuring 17cm by 9cm by 8cm.
Coordinate axes are taken as shown.



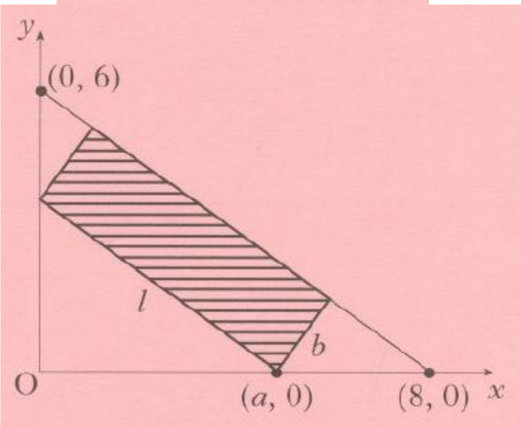
a) The point A has coordinates (0,9,8) and C has coordinates (17,0,8).
Write down the coordinates of B. (1)

b) Calculate the size of angle ABC. (6)

3) The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. the extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a,0)$.



a) i) Show that $l = \frac{5}{4}a$.

ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8 - a)$. (3)

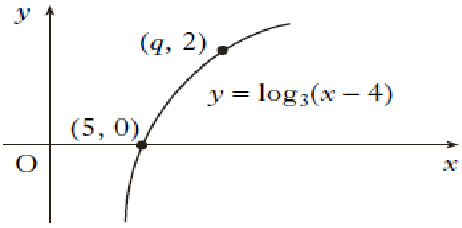
b) Find the value of a which produces the largest area of the extension. (4)

(19)

Logs & Exponentials

Questions 1 to 5 are non-calculator

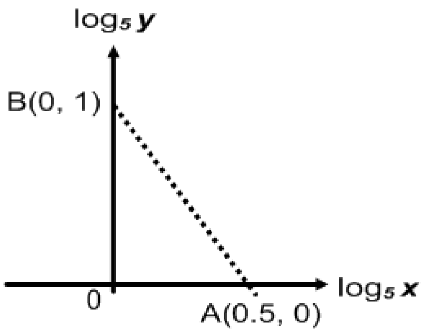
1) The diagram shows part of the graph of $y = \log_3(x - 4)$.
The point $(q, 2)$ lies on the graph.
What is the value of q? (2)



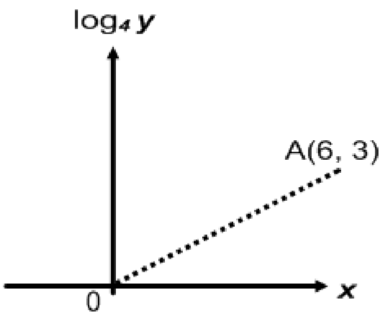
2) Evaluate $\log_5 2 + \log_5 50 - \log_5 4$ (3)

3) Solve $\log_4(5 - x) - \log_4(3 - x) = 2$, $x < 3$. (4)

4) The graph represents the law $y = kx^n$.
Find the values of k and n. (4)



5) Two variables x and y are connected by the law $y = a^x$.
Find the value of a. (4)



6) Before a forest fire was brought under control, the spread of the fire was described by a law of the form $A = A_0e^{kt}$ where A_0 is the area covered by the fire when it was first detected and A is the area covered by the fire t hours later.
If it takes one and a half hours for the forest fire to double, find the value of the constant k. (3)

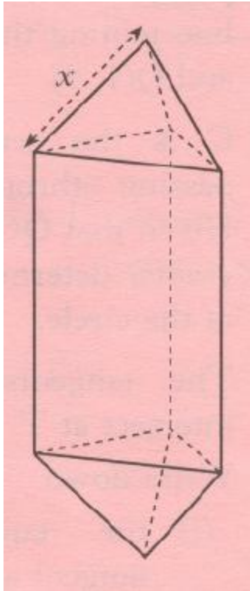
(20)

Mixed Exercise 8

1) A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.
The surface area, A, of the solid is given by

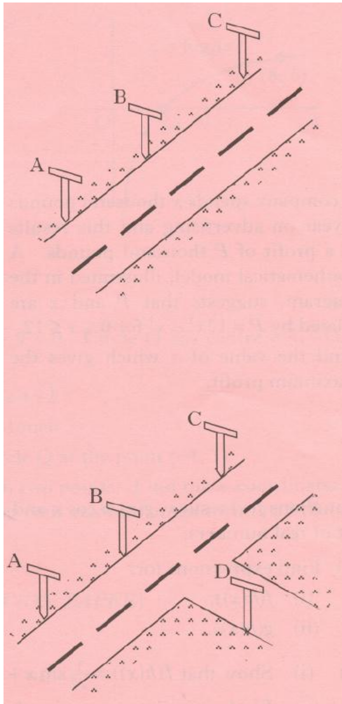
$$A(x) = \frac{3\sqrt{3}}{3}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.
Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. (6)



2) a) Road makers look along the tops of a set of T-rods to ensure that straight sections of road are being created. Relative to suitable axes the top left corners of the T-rods are the points A(-8,-10,-2), B(-2,-1,1) and C(6,11,5). Determine whether or not the section of road ABC has been built in a straight line. (3)

b) A further T-rod is placed such that D has coordinates (1,-4,4). Show that DB is perpendicular to AB. (3)



3) Find x if $4\log_x 6 - 2\log_x 4 = 1$. (3)

(15)

Integration 1

1) Find

a. $\int x^3 dx$ (1) b. $\int 6x^2 dx$ (1) c. $\int -10x^4 dx$ (1)

2) Find

a. $\int 2x^{\frac{1}{2}} dx$ (1) b. $\int \frac{1}{3x^{\frac{1}{2}}} dx$ (2) c. $\int \frac{2}{3\sqrt[3]{x}} + \sqrt{x}\, dx$ (2) d. $\int \left(5x^2 + 5 - \frac{3}{2x^2}\right) dx$ (2)

3) Find $\int \frac{1}{(7-3x)^2}\, dx$ (2)

4) Find $\int \frac{(x^2-2)(x^2+2)}{x^2}\, dx,\ x \neq 0$ (4)

5) The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point (-1,9). Express y in terms of x. (4)

6) A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $(\frac{5}{12}\pi, \sqrt{3})$.
Find y in terms of x. (4)

7) Evaluate

a. $\int_{-1}^{1} (2x-1)\, dx$ (2) b. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 3cos2x\, dx$ (3) c. $\int_0^1 \frac{dx}{(3x+1)^{1/2}}$ (4)

8) a) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}, x < 2$. (2)

b) Hence write down $\int \frac{x^2}{(8-x^3)^{\frac{1}{2}}}\, dx$. (1)

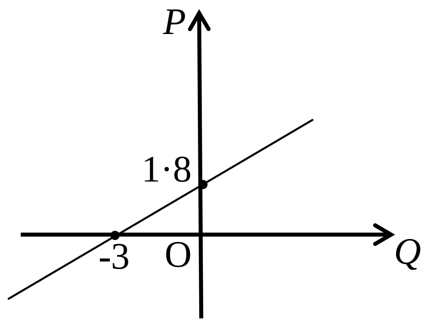
9) A point moves in a straight line such that its acceleration a is given by $a = 2(4-t)^{\frac{1}{2}}, 0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. (4)

(40)

Mixed Exercise 9

1) The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.
If $f'(x) = \sin(3x)$, express y in terms of x. (4)

2) The results of an experiment give rise to the graph shown.



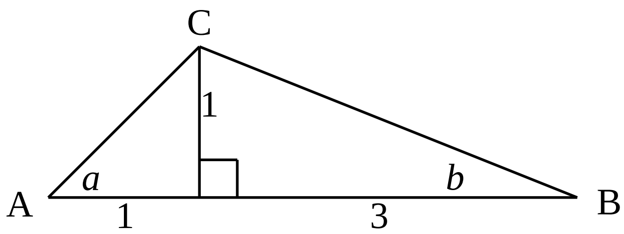
a) Write down the equation of the line in terms of P and Q. (2)

It is given that $P = \log_e p$ and $Q = \log_e q$.

b) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. (4)

3) The point Q divides the line joining P(-1,-1,0) to R(5,2,-3) in the ratio 2:1.
Find the coordinates of Q. (3)

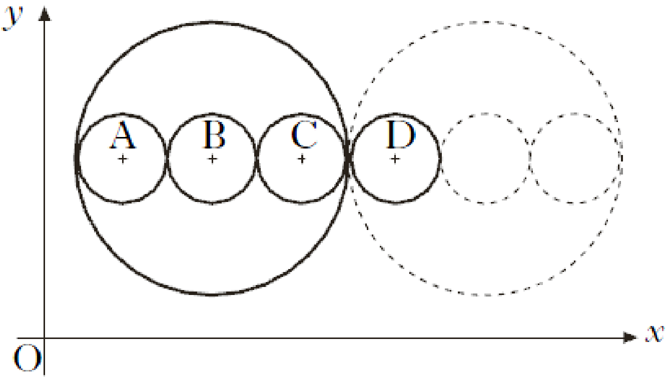
4) In triangle ABC, show that the exact value of sin(a + b) is $\frac{2}{\sqrt{5}}$. (4)



(17)

Circles

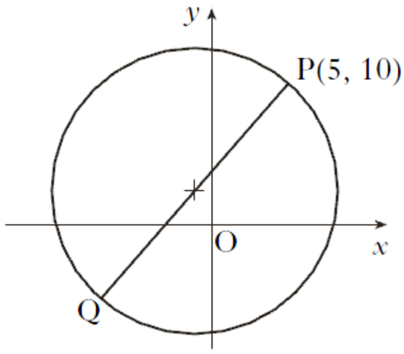
1) The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.
Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.
This pattern is continued, as shown in the diagram.
Find the equation of the circle with centre D. (5)



2) (*a*) Show that the point P(5, 10) lies on circle C_1 with equation $(x + 1)^2 + (y - 2)^2 = 100$. (1)

(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. (5)

(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q.
The radius of each of these circles is twice the radius of circle C_1.
Find the equations of circles C_2 and C_3. (4)



3) (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. (2)

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.
Find the distance between the centres of these two circles and hence show that the circles intersect. (4)

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.
Find the coordinates of the points of intersection of the two circles. (5)

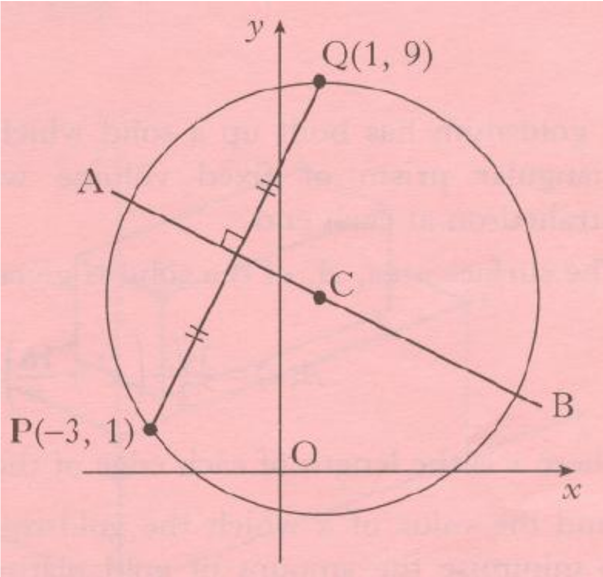
(26)

Mixed Exercise 10

1) a) Find the equation of AB, the perpendicular bisector of the line joining the points P(-3,1) and Q(1,9). (4)

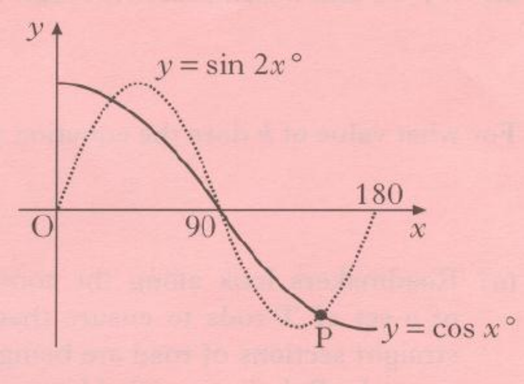
b) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. (3)

c) The tangents at P and Q intersect at T.
Write down
i) the equation of the tangent at Q.
ii) the coordinates of T. (2)



2) a) Solve the equation sin $2x°$ – cos $x°$ = 0 in the interval $0 \le x \le 180$. (4)

b) The diagram shows part of two trigonometric graphs, $y = \sin 2x°$ and $y = \cos x°$.
Use your solutions in (a) to write down the coordinates of the point P. (1)



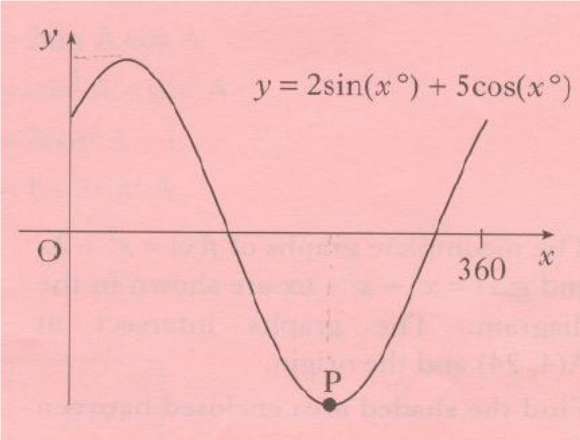
3) A curve for which $\frac{dy}{dx} = 3\sin(2x)$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$.
Find y in terms of x. (4)

(18)

The Wave Function

1) Express $8 \cos x° - 6 \sin x°$ in the form $k \cos(x + a)°$ where $k > 0$ and $0 < a < 360$. (4)

2) Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.



a) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \leq a \leq 360$. (4)

b) Find the coordinates of the minimum turning point P. (3)

3) a) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \leq a \leq 90$. (4)

b) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \leq x \leq 90$. (3)

(18)

Mixed Exercise 11

1) Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre (-2,-1) and radius $2\sqrt{2}$.

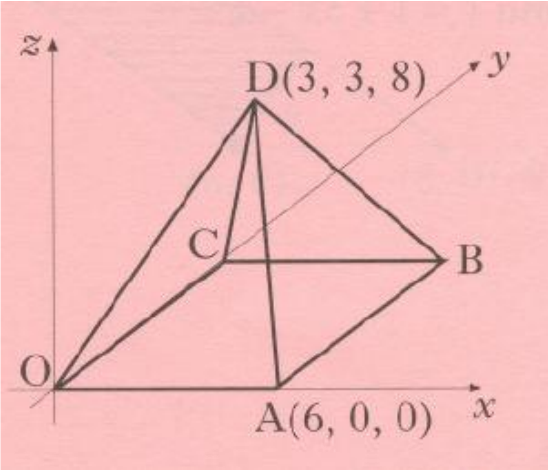
a) i) Show that the radius of the circle P is $4\sqrt{2}$.

ii) Hence show that circles P and Q touch. (4)

b) Find the equation of the tangent to circle Q at the point (-4,1). (3)

c) The tangent in (b) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing your answer in the form $a \pm b\sqrt{3}$. (3)

2) The diagram shows a square-based pyramid of height 8 units. Square OABC has a side length of 6 units. The coordinates of A and D are (6,0,0) and (3,3,8).
C lies on the y-axis.

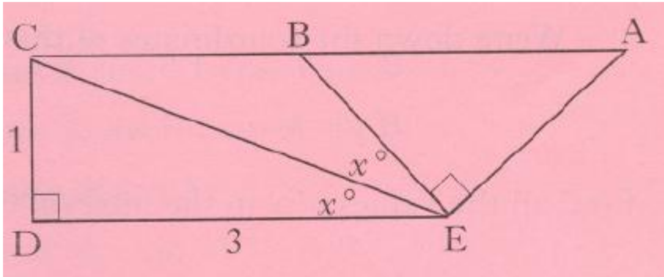


a) Write down the coordinates of B. (1)

b) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. (2)

c) Calculate the size of angle ADB. (4)

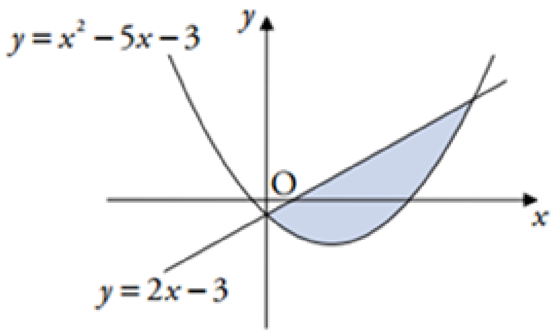
3) In the diagram
angle DEC = angle CEB = x° and
angle CDE = angle BEA = 90°.
CD = 1 unit; DE = 3 units.
By writing angle DEA in terms of x°, find the exact value of cos(DEA). (7)



(24)

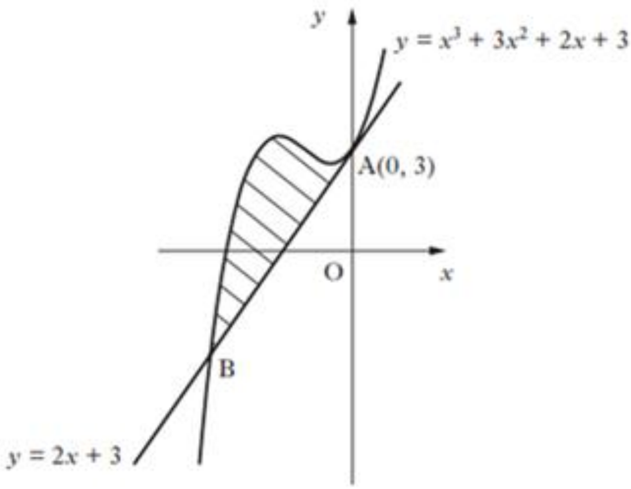
Integration 2

1) The diagram shows the line with equation $y = 2x - 3$ and the curve with equation $y = x^2 - 5x - 3$.
Write down the integral which represents the shaded area.
Do not carry out the integration. (3)

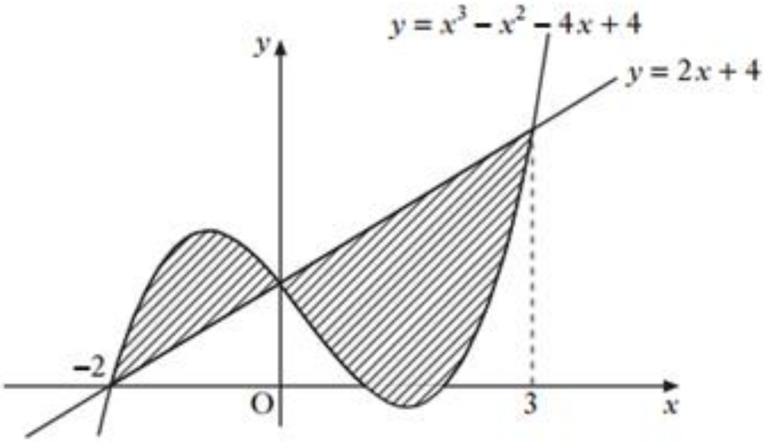


2) Given that $\int_4^t (3x + 4)^{-\frac{1}{2}}\, dx = 2$, find the value of t. (5)

3) The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A (0, 3) as shown in the diagram.
The line meets the curve again at B.
Show that B is the point (-3, -3) and find the area enclosed by the line and the curve. (6)



4) The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.
The curve and the line intersect at the points (-2, 0), (0, 4) and (3, 10).



Calculate the total shaded area. (10)

(24)

Mixed Exercise 12

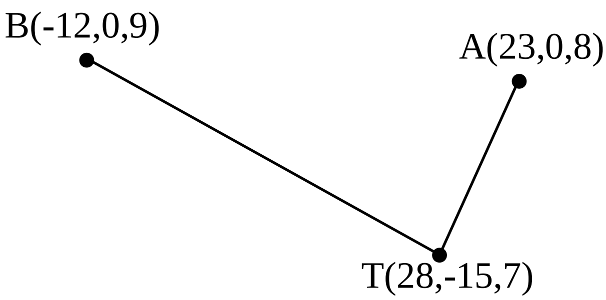
1) The point P(2,3) lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. (4)

2) The amount A_t micrograms of a certain radioactive substance remaining after t years decreases according to the formula $A_t = A_0\, e^{-0.002t}$, where A_0 is the amount present initially.

   a) If 600 micrograms are left after 1000 years, how many micrograms were present initially? (3)

   b) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance? (4)

3) The sketch shows the positions of Andrew (A), Bob (B) and Tracy (T) on three hill-tops.
Relative to a suitable origin, the coordinates (in hundreds of metres) of three people are A(23,0,8), B(-12,0,9) and T(28,-15,7).
In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



   a) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. (2)

   b) Calculate the angle between these two beams. (5)

(18)

Recurrence Relations

1) a) A sequence is defined by $u_{n+1} = -\frac{1}{2} u_n$ with $u_0 = -16$.
Write down the values of u_1 and u_2. (1)

b) A second sequence is given by 4, 5, 7, 11,…..
It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.
Find the values of p and q. (3)

c) Either the sequence in (a) or the sequence in (b) has a limit.
i) Calculate this limit.
ii) Why does the other sequence not have a limit? (3)

2) The first three terms of a sequence are 4, 7 and 16.
The sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

Find the values of m and c. (4)

3) A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

a) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? (3)

b) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition? (3)

4) A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$.

a) Write down the condition on k for this sequence to have a limit. (1)

b) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of k. (3)

(22)