Advanced Higher Practice Paper (3 hours)

A1. (a) Find partial fractions for $\dfrac{9}{x^2-4}$

(b) Hence obtain $\int \dfrac{x^2}{x^2-4}$ dx

A2. Use the substitution $u = 7 - x^3$ to evaluate the integral

$$\int \frac{x^2}{7-x^3}dx$$

A3. Use Gaussian elimination to solve the system of linear equations

$x + y + z = 0$

$-y - z = 1$

$x + 2y + z = 15$

A4. Find $\dfrac{dy}{dx}$ at (1,2) if $x^2 + 3xy + y^2 = 11$

A5. A horizontal firing torpedo has an acceleration (until its fuel runs out) of

$a= 9 + 12t - 0.5t^2$

where a is the acceleration (m/s/s) and t is the time in seconds

(a) Obtain a formula for its speed, t seconds after firing
(b) The torpedo has enough fuel for 7 seconds. What horizontal distance would it have covered when it ran out of fuel?

A6. Let $u_1, u_2, u_3 \ldots u_n$ be an arithmetic sequence and v_1, v_2, v_3, v_n be a geometric sequence

$u_1=v_1=45$ and $u_5=v_3=5$

(a) Find u_{11}

(b) Give that the geometric sequence is a series of positive numbers calculate $\sum_{n=1}^{\infty} v_n$

A7. Use induction to prove $5^{2n-1} + 1$ is divisible by 6.

A8. Let z = $\cos\theta + i\sin\theta$

(a) Find the real and imaginary parts of z^6

(b) Use de Moivre's theorem to write down an expression for z^6 in terms of θ

(c) Use your answers to (a) and (b) to express cos 6θ in terms of cos θ and sin θ

A9. Evaluate the following integral using partial fractions-

$$\int \frac{6x^2+20x+9}{x^3+2x^2+x}\,dx$$

A10. Let the function f be given by $f(x) = 2^x \cos x$

(a) Where does the function cross the y axis?

(b) Find the stationary points.

(c) Sketch the graph of y=f(x)

B1. Use the Euclidean Algorithm to find integers of x and y such that

$408x + 126y = 6$

B2. (a)Show that $\frac{d}{dx}(ln(lnx)) = \frac{1}{x\,ln\,x}$ and hence solve the differential equation $x\,lnx\,\frac{dy}{dx} + y = 2\,lnx$ given that $y = 2$ when $x = e$.

(b) Show , by means of the substitution $x = e^t$, that the differential equation

$$x^2\frac{d^2y}{dx^2} - 6x\frac{dy}{dx} + 6y = x^2 + ln\,x$$

reduces to $\frac{d^2y}{dt^2} - 7\frac{dy}{dt} + 6y = e^{2t} + t$.

Hence find the general solution of y in terms of x.

B3. Use Maclaurin's theorem to write down the expansions, as far as x^3, of

(i) e^{4x}
(ii) $(1-x)^{-3}$

B4. For the matrix $A = \begin{pmatrix} -1 & 4 & 1 \\ 2 & -3 & 1 \\ 1 & 2 & -1 \end{pmatrix}$

(a) Show that it is invertible ie |A|=0
(b) Find A^{-1}

(c) Solve the matrix equations AX=B where $x = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ and $B = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$