Variation

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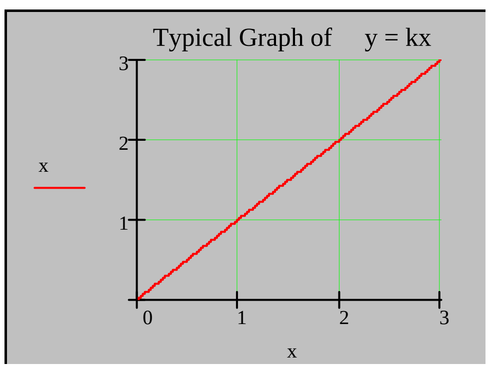
Graduate Bsc (Hons) MathsSci (Open) GIMA

There are 2 types of variation namely DIRECT and INDIRECT.
Direct variation: - This is when two quantities that are related to each other behave "in the same way" in the sense that if one is increasing/decreasing the other is increasing/decreasing.

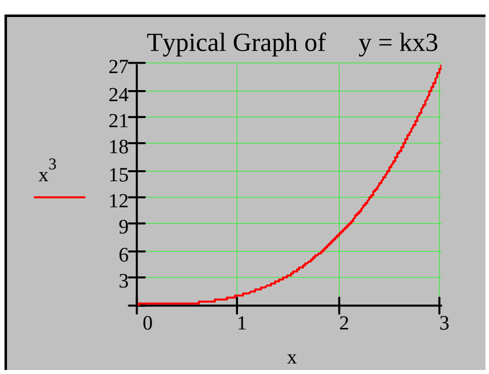
e.g. $y \alpha x$

This means that as x increases/decreases y increases/decreases.
Typical equation is $y = kx$ where k is just a constant number.



e.g. $y \alpha x^3$
This means that as x increases/decreases y increases/decreases by the cube of x.

Typical equation $y = kx^3$ where k is just a constant number



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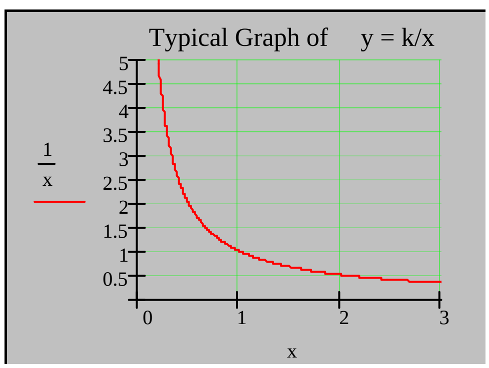
Indirect variation :- This is when two quantities that are related to each other vary "in the opposite way" to each other. If one is increasing/decreasing the other is decreasing/increasing.

e.g. $y \propto 1/x$

This means that as x increases/decreases y decreases/increases.

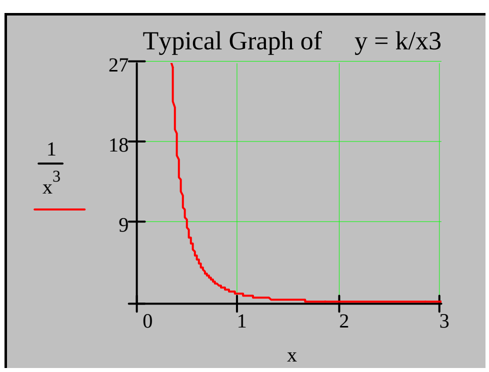
Typical equation is $y = k/x$ where k is just a constant number.



E.g. $y \propto 1/x^3$

This means that as x increases/decreases y increases/decreases by the 1/cube of x.

Typical equation $y = k/x^3$ where k is just a constant number



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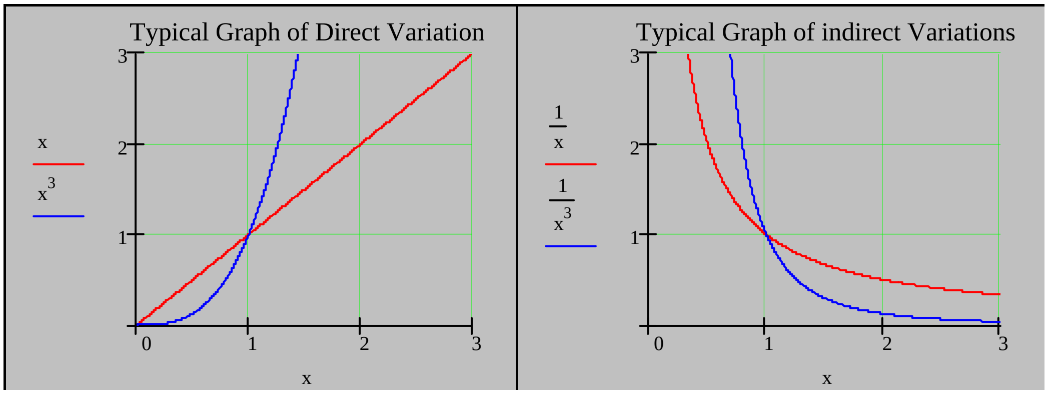
Comparing the rate of variation, the direct variations and indirect variations above are plotted together both.



We can know combine both these types of variation to create more complicated relationships between a given quantity and the factors that influence that quantity.

e.g. In physics it is well known that the following is true :-

force is directly proportional to mass (m) and also to acceleration (a)

$$F \propto m \text{ and } \quad F \propto a$$

this gives the equation

$$F = k \cdot m \cdot a \qquad \text{In this case k is 1}$$

$$F = m \cdot a$$

What happens to F if the mass (m) is doubled. we now have 2(m) instead of (m) hence

$$F = (2m) \cdot a = 2(m \cdot a)$$

Hence F is double the original value.

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What happens to F if the mass (m) is doubled. and the acceleration (a) is also doubled. We now have 2(m) instead of (m) and 2(a) instead of (a) hence

$$F = (2m)\cdot(2a) = 4(m\cdot a)$$

Hence F has now increased by a factor of 4 from the original value. Finally a more complicated example but the same rules apply.

Give the formula below determine how (m) varies with each quantity

$$m = \frac{k\cdot x\cdot y^2}{z^3}$$

k is just a constant number

Solution
m directly varies with x
m directly varies with squares of y
m indirectly varies with the cube of z

If we double y and double z what happens to m

Solution
We now have 2y instead of y and 2z instead of z hence we have

$$m = \frac{k\cdot x\cdot(2y)^2}{(2z)^3} = \frac{k\cdot x\cdot 4(y)^2}{8z^3} = \frac{4k\cdot x(y)^2}{8z^3} = \frac{1}{2}\cdot\left(\frac{k\cdot x\cdot y^2}{z^3}\right)$$

Hence m is half the original value.