Trigonometry

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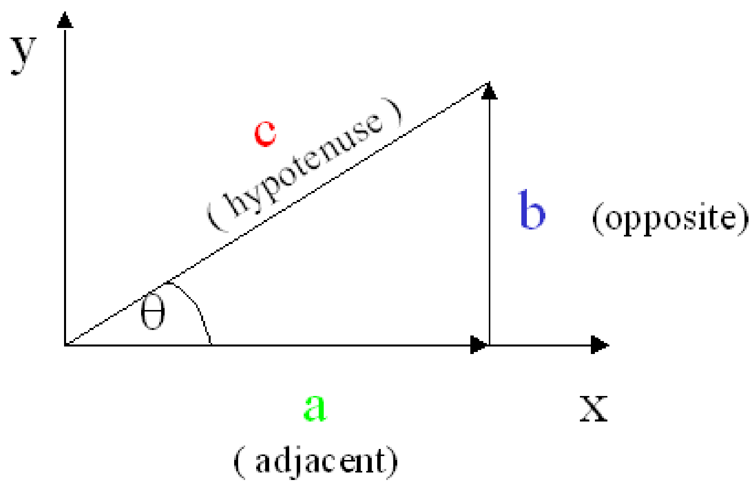
The Keypoints you need to know for Standard Grade Mathematics are as follows:-

Right-Angle Triangles.
Sine, cosine, tan and associated graphs.
Area of Triangles.
Sine Rule.
Cosine rule.
Square of sine plus square of cosine equals one.



Right - Angle Triangle (RAT)

$c^2 = a^2 + b^2$

The sine of the angle θ is given by the ratio:-

$$\sin(\theta) = \frac{b}{c} = \frac{opp}{hyp}$$

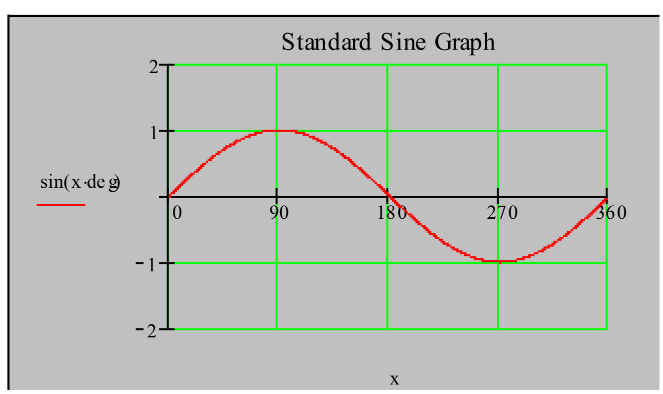
Similarly the cosine of the angle θ is given by the ratio:-

$$\cos(\theta) = \frac{a}{c} = \frac{adj}{hyp}$$

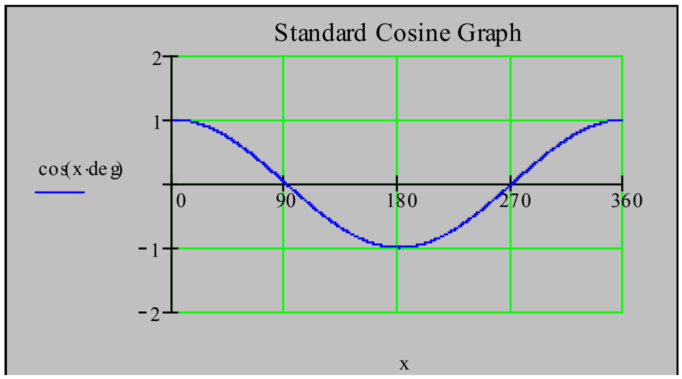
And tangent of the angle θ is given by the ratio:-

$$\text{tangent}(\theta) = \frac{b}{a} = \frac{opp}{adj}$$

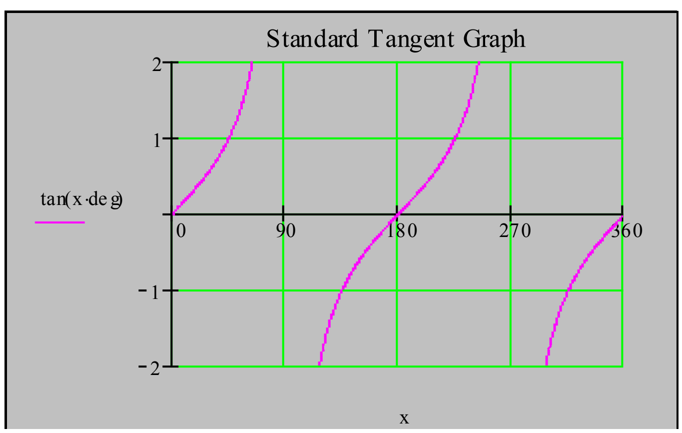
If we plot the angle θ along the x-axis and the ratio (opp/hyp) on y-axis we get the graph of the Standard Sine:-



Similarly if we plot the angle θ along the x-axis and the ratio (adj/hyp) on the y-axis we get the graph of the Standard Cosine:-



And if we plot the angle θ along the x-axis and the ratio (opp/adj) on the y-axis we get the graph of the Standard Tangent:-



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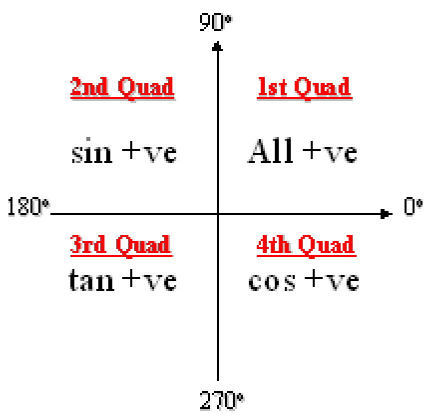
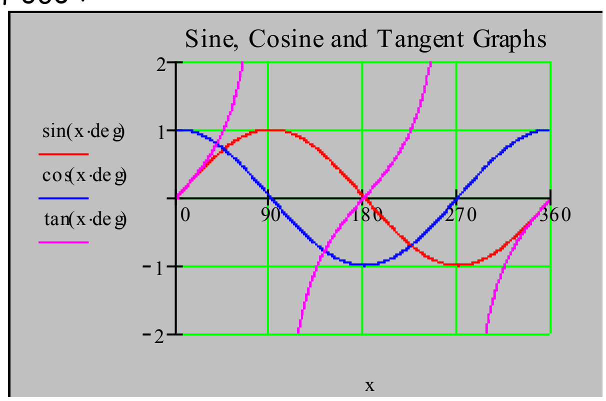
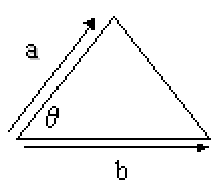
From the sine, cosine and tangent graphs we can summarises where their values are positive and negative over the range of 360°:-



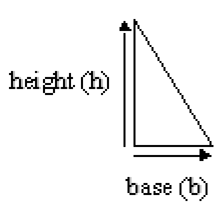


The area of any Triangle is given by the formula:-



If you know 2 sides and the angle between them you can use

$$A = \frac{1}{2} \cdot a \cdot b \cdot \sin\theta$$



For right-angle triangle $A = \frac{1}{2} \cdot \text{base} \cdot \text{height}$

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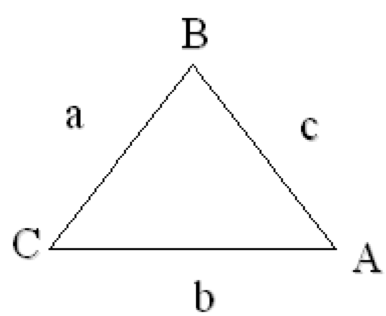
E.g. Find the area of the triangle above for a = 10, b = 8 and θ = 30°.

$$A = \frac{1}{2}\cdot b\cdot h\cdot \sin(\theta) \qquad A = \frac{1}{2}\cdot 10\cdot 8\cdot \sin\left(30^{o}\right) \qquad A = \frac{1}{2}\cdot 10\cdot 8\cdot \sin\left(30^{o}\right) = (0.5)\cdot 80\cdot (0.5) = 20$$

The Sine Rule

The sine rule is:-

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$



You can use the sine rule when you need to find a length or angle when you have two pairs of opposites with one unknown.

E.g. Find the angle C in the diagram above given A=30°, a = 6 and c = 6

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \qquad \frac{6}{\sin 30^{o}} = \frac{6}{\sin C^{o}} \qquad \sin C = \frac{6\cdot \sin 30^{o}}{6}$$

$$\sin C = \frac{6\cdot(0\cdot 5)}{6} = 0.5 \qquad C = \sin^{-1}(0.5) = 30^{o}$$

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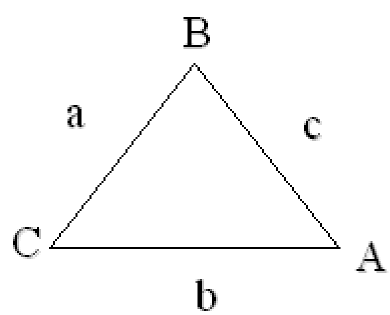
The Cosine Rule

The cosine rule has three formats all equally valid:-

$a^2 = b^2 + c^2 - 2b \cdot c \cdot \cos(A)$ $\quad b^2 = a^2 + c^2 - 2a \cdot c \cdot \cos(B)$ $\quad c^2 = a^2 + b^2 - 2a \cdot b \cdot \cos(C)$



You can use the cosine rule when you need to find a length or angle when you know two lengths and the angle between them.

E.g. Find the length (a) in the diagram above given A=60°, b = 6 and c = 6

$a^2 = b^2 + c^2 - 2b \cdot c \cdot \cos(A)$ $\quad a^2 = 6^2 + 6^2 - 2 \cdot 6 \cdot 6 \cdot \cos(60^o)$ $\quad a^2 = 72 - 72 \cdot \cos(60^o)$

$a^2 = 72 - 72(0.5)$ $\quad a^2 = 36$ $\quad a = 6$

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Sketching Sine and Cosine graphs

To sketch graphs of the following type :-

$y = k + A\sin(b \cdot x - \alpha^o)$ Or $y = k + A\cos(b \cdot x - \alpha^o)$

We simply start from the basic sine or cosine graph and move it according to the following rules:-

If k is positive/negative move the sine/cosine graph by k units in the positive/negative y-axis direction.

If A > 1 then stretch the sine/cosine graph by a factor of A in the y-axis direction.

If 0 < A < 1 then squash the sine/cosine graph by a factor of A in the y-axis direction.

If A has a negative sign then flip the graph about the x-axis then stretch/squash as above in the y direction.

If α has a negative sign in front of it then move the sine/cosine graph α^o in the positive x-axis direction.
If α has a positive sign in front of it then move the sine/cosine graph α^o in the negative x-axis direction.

If b > 1 then stretch squash the sine/cosine graph by a factor of b in the x-axis direction.

If 0 < b < 1 then stretch the sine/cosine graph by a factor of b in the x-axis direction.

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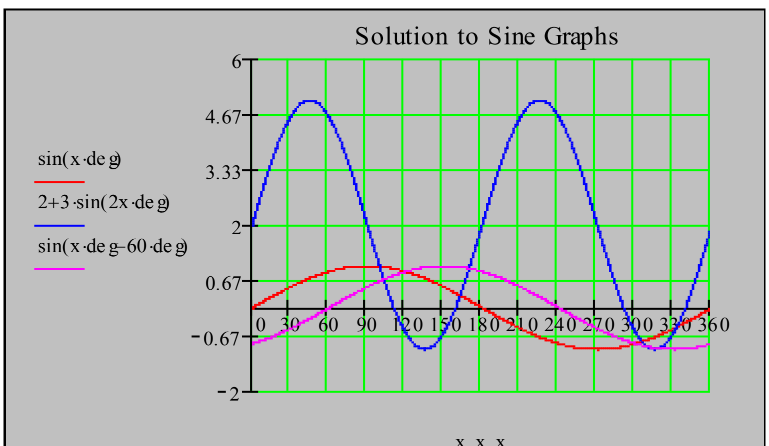
Some examples

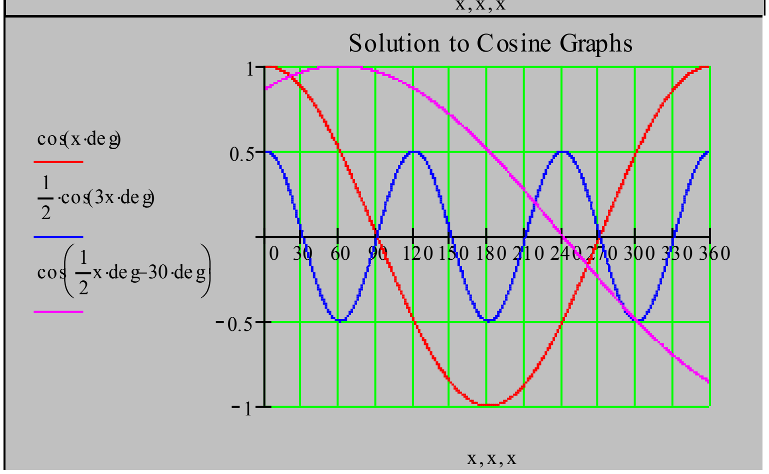
Sketch the following graphs

$y = 2 + 3 \cdot \sin(2 \cdot x)$ $\quad$ $y = (0.5)\cos(3 \cdot x)$ $\quad$ $y = \sin\left(x - 60^0\right)$ $\quad$ $y = \cos\left(\frac{1}{2} \cdot x - 30^0\right)$

Sine functions are shown with respect to the Standard Sine.

Cosine functions are shown with respect to the Standard Cosine.





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Finally it is worth remembering the following result below which is valid for all angles.

(Convince yourself by trying calculations on your calculator).

$$\sin^2 x + \cos^2 x = 1$$

Shown graphically it is as follows