Straight Line Theory

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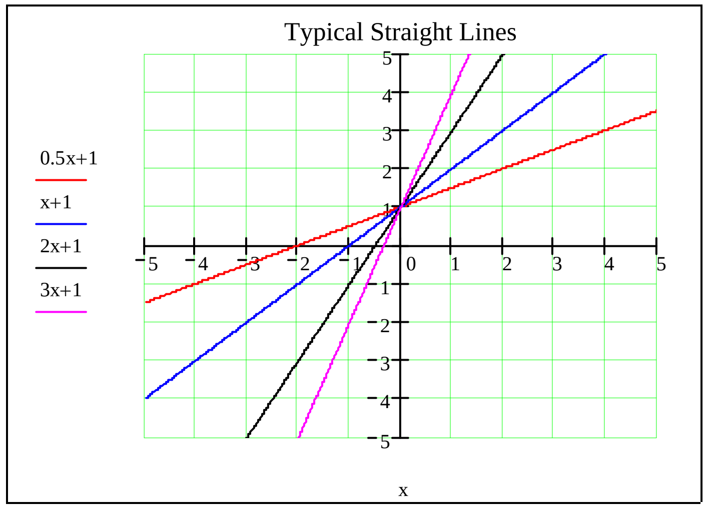
Standard form for a straight line is

$y = mx+c$ or $y-b = m(x-a)$

m = gradient

(a,b) = point on the line

c = value where line crosses y-axis



1. To find the equation of a line you need to know:-

   Two points $A (x_1, y_1)$ and $B (x_2, y_2)$

   or

   One point and the gradient (m)

   $A (x_1, y_1)$ and (m)

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2. Gradient (m) is found by evaluating

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

3. A straight line crosses x-axis at (x,0) and the y-axis at (0,C) .

4. Two lines are perpendicular (at right angles) to each other if:-

$$m_1 \cdot m_2 = -1$$

5. The point of intersection of 2 lines can be found by solving:-

Line one = Line two

$$Y_1 = Y_2$$

$$m_1 \cdot x_1 + C_1 = m_2 \cdot x_2 + C_2$$

6. The angle between the line and the x-axis is given by

$\theta = \tan^{-1}(m)$ m =gradient

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1. Find the equation (1) of the line that passes through the points A (1, 4) and B (2, 6).

Also find the equation (2) of the line with gradient m = - 0.5 and passing through the point (2,-1).

Solution

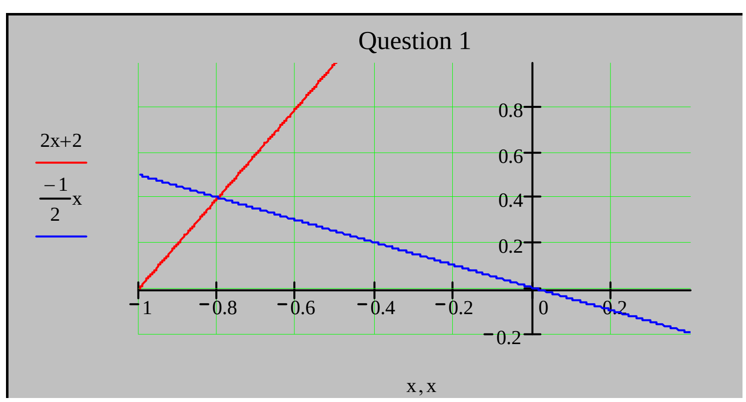
Equation (1)

$$m_1 = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(6-4)}{(2-1)} = 2$$

Choosing point B (2, 6)

$$c_1 = y - m \cdot x = 6 - 2 \cdot (2) = 2$$

Hence equation (1) is $y = 2 \cdot x + 2$



Equation (2)

$$m_2 = -0.5$$

Point given is (2,-1)

$$c_2 = y - m \cdot x = -1 - (-0.5) \cdot 2 = 0$$

Hence equation (2) is $y = \frac{-1}{2} \cdot x$

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2. Find the points were the equations (1) and (2) cut the x and y axises.

Solution

Equation (1)

When x = 0 then y = c = 2; Hence y-axis is cut at (0, c) = (0, 2)

When y = 0 then

$$0 = 2 \cdot x + 2$$

$$x = \frac{-2}{2} = -1$$

Hence x-axis is cut at (-1, 0)

Equation (2)

When x = 0 then y = c = 0; Hence y-axis is cut at (0, c) = (0, 0)

When y = 0 then

$$0 = \frac{-1}{2} \cdot x$$

$$x = \frac{2 \cdot 0}{-1} = 0$$

Hence x-axis is cut at (0, 0)

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3. Show that the 2 lines are perpendicular to each other.

Solution

If perpendicular to each other then the following is true.

$$m_1 = 2$$

$$m_1 \cdot m_2 = -1$$

$$m_2 = \frac{-1}{2}$$

$$2 \cdot \left(\frac{-1}{2} \right) = -1$$

Hence lines are perpendicular.

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4. Find the point where the lines intersect.

Solution

Where both lines intersect we have.

$$\text{line}_1 = \text{line}_2$$

$$m_1 \cdot x_1 + c_1 = m_2 \cdot x_2 + c_2$$

$$2 \cdot x + 2 = \left(\frac{-1}{2} \cdot x + 0\right)$$

At the point of intersect!

$$x_1 = x_2$$

$$2 \cdot x + \frac{1}{2} \cdot x = -2 \qquad \frac{5}{2} \cdot x = -2 \qquad x = \frac{-4}{5}$$

Substituting $x = \frac{-4}{5}$

Into one of the original equations ((1) or (2) it does not matter which one as either one is equally valid) we get:

Equation (1) $y = 2 \cdot x + 2$

$$y = 2 \cdot \left(\frac{-4}{5}\right) + 2 = \frac{-8}{5} + \frac{10}{5} = \frac{2}{5}$$

Point is $\left(\frac{-4}{5}, \frac{2}{5}\right)$

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Equation (2) $y = \frac{-1}{2} \cdot x$

$$y = \frac{-1}{2} \cdot \left(\frac{-4}{5}\right) = \frac{4}{10} = \frac{2}{5}$$

Point is $\left(\frac{-4}{5}, \frac{2}{5}\right)$

5. Find the angle made by line (1) and the positive x-axis and repeat for line (2).

Solution

$$\theta_1 = \tan^{-1}(m_1) = \tan^{-1}(2) = 63.4^0$$

$$\theta_2 = \tan^{-1}(m_2) = \tan^{-1}\left(\frac{-1}{2}\right) = 180^0 - 26.6^0 = 153.4^0$$

Second quadrant!