Simultaneous Equations

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When we have 2 equations with 2 unknown variables in them, we call the equations Simultaneous Equations. There is a set method for solving them.

$2x + 4y = 20$ Equation 1 with 2 variables x and y

$6x + 5y = 15$ Equation 2 with 2 variables x and y

We can solve these equations by eliminating one of the variables by using the simple rules of arithmetic and then substituting the variable found into one of the original equations to find the value of the variable that was eliminated.

Sounds all rather complicated but is actually very simple once you have solved a few. The method can be easily extended to 3 variables with 3 equations and beyond.

E.g. Given the 2 equations below find the values for p and g.

$4p + 3g = 130$ Equation (1)

$2p + 4g = 120$ Equation (2)

Step 1: If we multiply equation (2) by 2 we get

$4p + 3g = 130$ Equation (1)

$4p + 8g = 240$ New Equation (2)

Step 2: If we now subtract equation (1) from equation (2) we get

$$(4p - 4p) + (8g - 3g) = (240 - 130)$$

$$5g = 110 \qquad g = \frac{110}{5} = 22$$

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Step 3: Since we know the value of g we can now substitute this value into the original equation (1) or (2) to find p.

Choosing equation (2) we get

$$2p + 4g = 120$$
$$2p + (4 \cdot 22) = 120$$
$$2p = 120 - 88$$
$$2p = 32$$
$$p = 16$$

We can check that these values are correct by substituting the values into equation (1).

$$(4 \times 16) + (3 \times 22) = 130$$
$$(64) + (66) = 130$$
$$130 = 130$$

Note: that when you are solving simultaneous equations you are using various mathematical ideas and rules, for example we use the basic rules of arithmetic to rearranging equations.

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To get the graphic solution we can rearrange the two equations into the form (g =) or (p =) and then plot them on the same graph and where they intersect gives the solution. Choosing the form (g=) we have

Equation 1

$4p + 3g = 130$

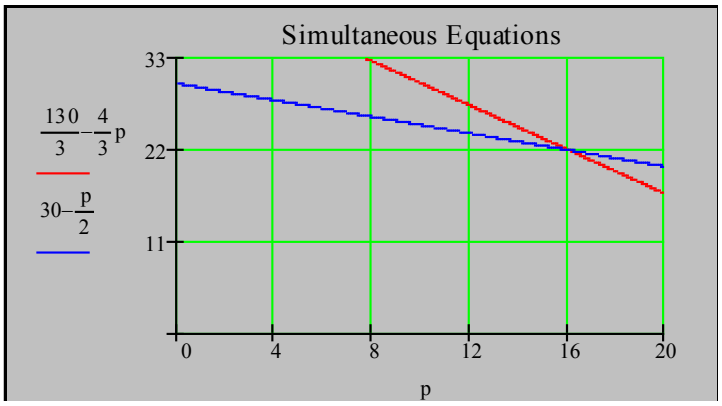
$$g = \frac{130}{3} - \frac{4p}{3}$$

Equation 2

$2p + 4g = 120$

$$g = \frac{120}{4} - \frac{2p}{4}$$

$$g = 30 - \frac{p}{2}$$



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Exercises on Simultaneous Equations

1. Solve the following set of equations.

$m + r = 180$ (1)

$9m + 7.5r = 1500$ (2)

Solution

Step 1: Multiply first equation by 7.5 we get

$7.5m + 7.5r = 1350$

Step 2: Subtract the equation in step 1 from equation (2)

$9m + 7.5r = 1500$
$7.5m + 7.5r = 1350$

$1.5m = 150$

$m = \frac{150}{1.5} = 100$

Step 3: Substitute value for (m) found in Step 2 into any of the original equations to find (r).

Choosing equation (1) as this is the easy option we get

$m = 100$ $100 + r = 180$ $r = 80$

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Check the values found for (m) and (r) by substituting them in equation (2). They should satisfy the equation (if not you have made an error!)

$9m + 7.5r = 1500$ $\quad 9\cdot(100) + 7.5(80) = 1500$ Correct!

Q2. Solve

$3x + 50y = 88.5$ Equation (1)

$4x + 60y = 133$ Equation (2)

Step 1: Multiply equation (1) by 4 and then multiply equation (2) by 3.

$12x + 200y = 354$ Equation (3)

$12x + 180y = 399$ Equation (4)

Step 2: We can now subtract equation (4) from equation (3) to eliminate (x).

$20y = 354 - 399 = -45$ $\quad y = \frac{-45}{20} = -2.25$

Step 3: Substitute value for (y) found in Step 2 into any of the original equations.

Choosing equation (1) we get

$3x + 50y = 88.5$ $\quad 3x - 112.5 = 88.5$ $\quad x = \frac{201}{3} = 67$

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To check those results are right, substitute (x) and (y) values into equation (2).

$4x + 60y = 133$

$4 \times 67 + 60 \times (-2.25) = 133$

$268 - 135 = 133$

$133 = 133$ Correct !

$268 - 135 = 133$

Q3. Solve

$5x + 3y = 1$ Equation (1)

$3x - 2y = 12$ Equation (2)

Step 1: Multiply equation (1) by 3 and then multiply equation (2) by 5.

$15x + 9y = 3$ Equation (3)

$15x - 10y = 60$ Equation (4)

Step 2: We can now subtract equation (4) from equation (3) to eliminate (x).

$9y - (-10y) = 3 - 60 = -57$

$y = \frac{-57}{19} = -3$

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Step 3: Substitute value for (y) found in Step 2 into any of the original equations.

Choosing equation (2) we get

$3x - 2\cdot(-3) = 12$ $\quad 3x = 12 - 6 = 6$ $\quad x = \frac{6}{3} = 2$

To check those results are right, substitute (x) and (y) values into equation (1).

$5(2) + 3(-3) = 1$ Correct!

Q4. Solve

$2x + y = 5$ Equation (1)
$x - 3y = 6$ Equation (2)

Step 1: Multiply equation (2) by 2.

$2x + y = 5$ Equation (1)
$2x - 6y = 12$ Equation (4)

Step 2: We can now subtract equation (4) from equation (1) to eliminate (x).

$y - (-6y) = y + 6y = 7y = -7$ $\quad y = \frac{-7}{7} = -1$

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Step 3: Substitute value for (y) found in Step 2 into any of the original equations.

Choosing equation (2) we get

$x - 3\cdot(-1) = x + 3 = 6 \qquad x = 6 - 3 = 3$

To check those results are right, substitute (x) and (y) values into equation (1).

$2(3) + (-1) = 5$ Correct!

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Q5. Solve

$3x - 2y = 11$ Equation (1)

$2x + 5y = 1$ Equation (2)

Step 1: Multiply equation (1) by 2 and equation (2) by 3.

$6x - 4y = 22$ Equation (3)

$6x + 15y = 3$ Equation (4)

Step 2: We can now subtract equation (4) from equation (3) to eliminate (x).

$-4y - 15y = -19y = 19$ $\qquad y = \frac{19}{-19} = -1$

Step 3: Substitute value for (y) found in Step 2 into any of the original equations.

Choosing equation (2) we get

$2x + 5(-1) = 2x - 5 = 1$ $\qquad x = \frac{6}{2} = 3$

To check those results are right, substitute (x) and (y) values into equation (1).

$3(3) - 2(-1) = 11$ Correct!

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Q6. Solve

$3x + 2y = 8.6$ Equation (1)

$5x + 3y = 13.6$ Equation (2)

Step 1: Multiply equation (1) by 5 and equation (2) by 3.

$15x + 10y = 43$ Equation (3)

$15x + 9y = 40.8$ Equation (4)

Step 2: We can now subtract equation (4) from equation (3) to eliminate (x).

$y = 43 - 40.8 = 2.2$

Step 3: Substitute value for (y) found in Step 2 into any of the original equations.

Choosing equation (2) we get

$5x + 3(2.2) = 5x + 6.6 = 13.6$ $\quad 5x = 13.6 - 6.6 = 7$ $\quad x = \frac{7}{5} = 1.4$

To check those results are right, substitute (x) and (y) values into equation (1).

$3(1.4) + 2(2.2) = 8.6$ Correct!