Quadratic Theory

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Quadratic Theory is a very important part of the Higher Still course and in Mathematics generally as it seems to pop up everywhere!

A polynomial function has the format

$a \cdot x^n$ Where n is a positive non-negative whole number

The degree of the polynomial is given by the largest power of x.

$x^4 + x^3 + x^2$ Has degree 4

When the highest power is of degree 2 we give it a special name "quadratic equation ".

We can find the roots of a quadratic (where it cuts the x-axis) by either factorising (If this can be done easily) or by using the formula below.

$x = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a}$ And $x = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a}$ for $a \cdot x^2 + b \cdot x + c = 0$

The discriminant of a quadratic equation is defined as being

$$b^2 - 4ac$$

The discriminant tells us a lot of useful information about the roots. We can have three situations:-

$b^2 - 4ac > 0$ Means 2 real roots

$b^2 - 4ac < 0$ Means no real roots

$b^2 - 4ac = 0$ Means one real root

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1. State which of the following functions are polynomial functions justifying your answer, stating the degree of the function where appropriate.

$x^3 + x^2 + 2x + 1$

$x^{-2} + 1$

$x^3 + x^4 + \frac{1}{2}x^6 + 5$

Solution

$x^3 + x^2 + 2x + 1$	Polynomial of degree 3
$x^{-2} + 1$	Not polynomial since -2 is a negative whole number
$x^3 + x^4 + \frac{1}{2}x^6 + 5$	Polynomial of degree 6

2. Find the roots of the following quadratic functions below.

$x^2 - 5x + 6$

$x^2 - 20x + 100$

Solution

$x^2 - 5x + 6 = (x - 3)(x - 2) = 0 \quad x = 3 \quad x = 2$

$x^2 - 20x + 100 = (x - 10)(x - 10) = 0 \quad x = 10$

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3. Find a quadratic equation that satisfies the given roots.

$x = 5$ and $x = 8$

Solution

Hence we have

$(x-8)(x-5) = 0$

$x^2 - 13x + 40 = 0$

4. By finding the discriminant of the quadratic equations below, determine the number roots each function has (you can evaluate them if you wish if you need the practice!)

$x^2 - 1 = 0$ $x^2 + 1 = 0$ $x^2 + 2x + 1 = 0$

Solution

$x^2 - 1 = 0$	$b^2 - 4ac = 0^2 - [4\cdot1\cdot(-1)] = 4 > 0$	Hence 2 real roots
$x^2 + 1 = 0$	$b^2 - 4ac = 0^2 - (4\cdot1\cdot1) = -4 < 0$	Hence no real roots
$x^2 + 2x + 1 = 0$	$b^2 - 4ac = (-2)^2 - (4\cdot1\cdot1) = 0$	Hence one real root