Quadratic Theory is a very important part of the Higher Still course and in Mathematics generally as it seems to pop up everywhere!

A polynomial function has the format

\[ a \cdot x^n \quad \text{Where } n \text{ is a positive non-negative whole number} \]

The degree of the polynomial is given by the largest power of \( x \).

\[ x^4 + x^3 + x^2 \quad \text{Has degree 4} \]

When the highest power is of degree 2 we give it a special name "quadratic equation".

We can find the roots of a quadratic (where it cuts the x-axis) by either factorising (If this can be done easily) or by using the formula below.

\[
x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{And} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad \text{for } a \cdot x^2 + b \cdot x + c = 0
\]

The discriminant of a quadratic equation is defined as being

\[ b^2 - 4ac \]

The discriminant tells us a lot of useful information about the roots. We can have three situations:-

\[ b^2 - 4ac > 0 \quad \text{Means 2 real roots} \]

\[ b^2 - 4ac < 0 \quad \text{Means no real roots} \]

\[ b^2 - 4ac = 0 \quad \text{Means one real root} \]
1. State which of the following functions are polynomial functions justifying your answer, stating the degree of the function where appropriate.

\[ x^3 + x^2 + 2x + 1 \]

\[ x^{-2} + 1 \]

\[ x^3 + x^4 + \frac{1}{2}x^6 + 5 \]

**Solution**

\[ x^3 + x^2 + 2x + 1 \] Polynomial of degree 3

\[ x^{-2} + 1 \] Not polynomial since \(-2\) is a negative whole number

\[ x^3 + x^4 + \frac{1}{2}x^6 + 5 \] Polynomial of degree 6

2. Find the roots of the following quadratic functions below.

\[ x^2 - 5x + 6 : \]

\[ x^2 - 20x + 100 \]

**Solution**

\[ x^2 - 5x + 6 = (x - 3)(x - 2) = 0 \quad x = 3 \quad x = 2 \]

\[ x^2 - 20x + 100 = (x - 10)(x - 10) = 0 \quad x = 10 \]
3. Find a quadratic equation that satisfies the given roots.

\[ x = 5 \quad \text{and} \quad x = 8 \]

**Solution**

Hence we have

\[ (x - 8)(x - 5) = 0 \]

\[ x^2 - 13x + 40 = 0 \]

4. By finding the discriminant of the quadratic equations below, determine the number roots each function has (you can evaluate them if you wish if you need the practice!)

\[ x^2 - 1 = 0 \quad x^2 + 1 = 0 \quad x^2 + 2x + 1 = 0 \]

**Solution**

\[ x^2 - 1 = 0 \quad b^2 - 4ac = 0^2 - [4 \cdot 1 \cdot (-1)] = 4 > 0 \quad \text{Hence 2 real roots} \]

\[ x^2 + 1 = 0 \quad b^2 - 4ac = 0^2 - (4 \cdot 1 \cdot 1) = -4 < 0 \quad \text{Hence no real roots} \]

\[ x^2 + 2x + 1 = 0 \quad b^2 - 4ac = (-2)^2 - (4 \cdot 1 \cdot 1) = 0 \quad \text{Hence one real root} \]