Mathematically Similar

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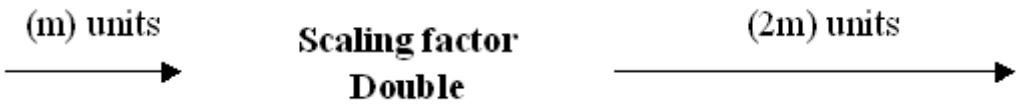
Mathematically similar in terms of Length, Area and Volume

When an object is enlarged or reduced while keeping all the lengths in proportion, the enlargement or reduction is said to be mathematically similar to the original object irrespective of its shape. In one dimension we are considering Length, in two dimensions Area and in three dimensions Volume.

In the following we consider what happens when we double the length in one, two and three dimensions. This is to show the idea of what is happening and for ease of understanding then we generalise the results at the end.

One Dimension: If we have length (m) units in the x-axis then if we double that length we simply get the length 2(m) units

The original length.



$\text{Length} = 2^1 \cdot m = 2 \cdot m$

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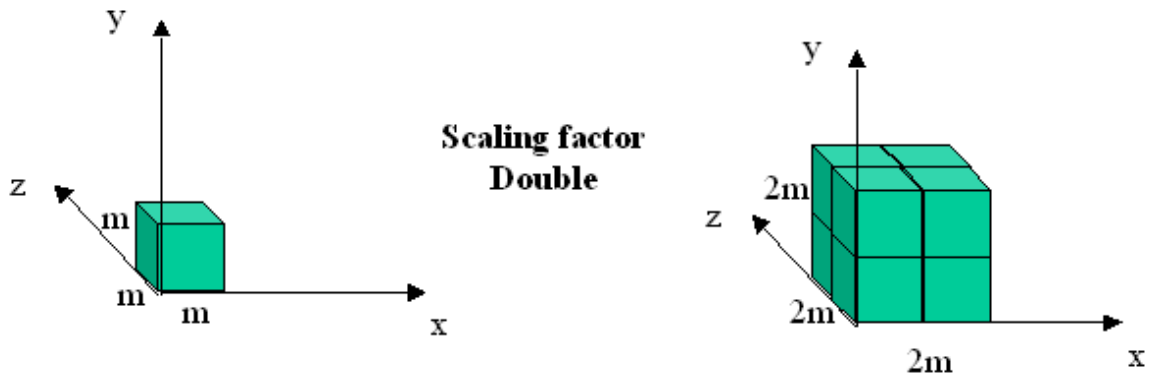
Two Dimensions: If we have two lengths each (m) units in the x and y-axis directions then the Area is given by:-

$\text{Area} = (2 \cdot m) \cdot (2 \cdot m) = 2^2 \cdot m = 4m^2$

Three Dimensions: If we have three lengths each (m) units in the x, y and z axises direction then the Volume is given by:-



$V = (2 \cdot m) \cdot (2 \cdot m) \cdot (2 \cdot m) = 2^3 \cdot m = 8m^3$

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We can see that if we enlarge an object in one or two or three dimensions by a factor of 2 the corresponding length, Area and Volumes are increased bya factor of:-

$$\text{Length} = 2^1 \qquad \text{Area} = 2^2 \qquad \text{Volume} = 2^3$$

This can be generalised so that for a given scaling factor (SF) if the objects are mathematically similar the resultant Length, Area or Volume will be

$$\text{Length} = SF \qquad \text{Area} = (SF)^2 \qquad \text{Volume} = (\text{SF})^3$$

Volume example: A company supply breakfast cereal in small, medium and large boxes. The small box has length 20 cm and volume of 600cm^3. The medium and large boxes have lengths 40 and 60cm respectively. Given they are mathematically similar, what are their volumes.

$$V_m = \left(\frac{40}{20}\right)^3 \times 600 = 4800cm^3$$

$$V_L = \left(\frac{60}{20}\right)^3 \times 600 = 16200cm^3$$

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Area example: A small football pitch has a length of 15m and its area is $150m^2$. If a full sized football pitch has a length of 60m, given they are mathematically similar, what is the area of the full sized pitch.

$$A_f = \left(\frac{60}{15}\right)^2 \times 150 = 2400m^2$$