

Trig. Past Papers Unit 2 Outcome 3

Written Questions

- [SQA] 1. Solve the equation $3 \cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \leq x \leq 360$. 5
- [SQA] 2. Solve the equation $\cos 2x^\circ + 5 \cos x^\circ - 2 = 0$, $0 \leq x < 360$. 5
- [SQA] 3. Find the exact solutions of the equation $4 \sin^2 x = 1$, $0 \leq x < 2\pi$. 4
- [SQA] 4. Solve the equation $2 \cos^2 x = \frac{1}{2}$, for $0 \leq x \leq \pi$. 3
- [SQA] 5. Solve the equation $\cos 2x^\circ + \cos x^\circ = 0$, $0 \leq x < 360$. 5
- [SQA] 6. Solve $2 \sin 3x^\circ - 1 = 0$ for $0 \leq x \leq 180$. 4
- [SQA] 7. 2
(a) Show that $2 \cos 2x^\circ - \cos^2 x^\circ = 1 - 3 \sin^2 x^\circ$. 2
(b) Hence solve the equation $2 \cos 2x^\circ - \cos^2 x^\circ = 2 \sin x^\circ$ in the interval $0 \leq x < 360$. 4
- [SQA] 8. Solve the equation $\sin 2x^\circ + \sin x^\circ = 0$, $0 \leq x < 360$. 5
- [SQA] 9. Find, correct to one decimal place, the value of x between 180 and 270 which satisfies the equation $3 \cos(2x^\circ - 40^\circ) - 1 = 0$. 5
- [SQA] 10. 3
(a) Write the equation $\cos 2\theta + 8 \cos \theta + 9 = 0$ in terms of $\cos \theta$ and show that, for $\cos \theta$, it has equal roots. 3
(b) Show that there are no real roots for θ . 1

- [SQA] 11. If $f(a) = 6 \sin^2 a - \cos a$, express $f(a)$ in the form $p \cos^2 a + q \cos a + r$.
Hence solve, correct to three decimal places, the equation $6 \sin^2 a - \cos a = 5$ for $0 \leq a \leq \pi$. 4

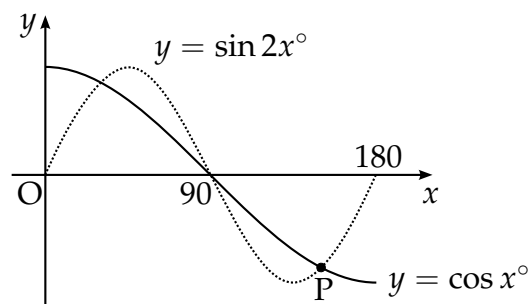
- [SQA] 12. Find the values of t , where $0 < t < 2\pi$, for which $4 \cos(2t - \frac{\pi}{4})$ has its maximum value. 4

- [SQA] 13. Solve the equation $2 \sin(2x - \frac{\pi}{6}) = 1$, $0 \leq x < 2\pi$. 4

- [SQA] 14. (a) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$. 4

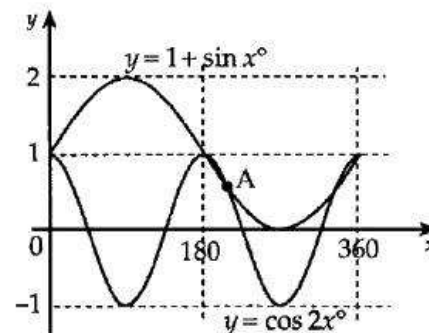
- (b) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (a) to write down the coordinates of the point P.



- [SQA] 15. The diagram shows two curves with equations $y = \cos 2x^\circ$ and $y = 1 + \sin x^\circ$ where $0 \leq x \leq 360$. 4

Find the x-coordinate of the point of intersection at A.



- [SQA] 16. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(a) Find expressions for:

- (i) $f(g(x))$;
(ii) $g(f(x))$.

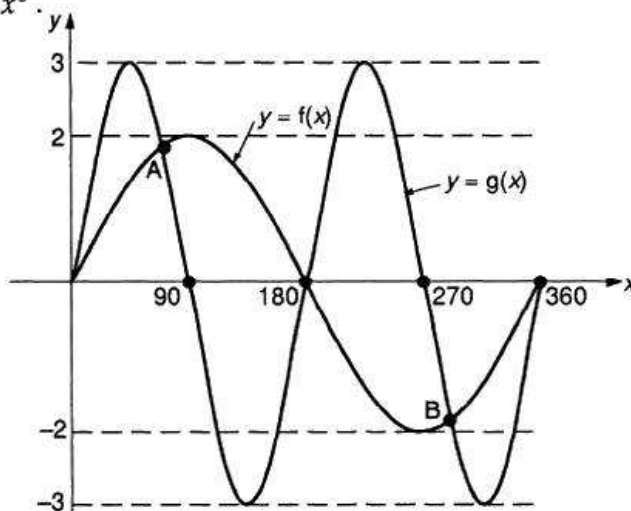
(b) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$.

[SQA] 17. (a) Solve the equation $3\sin 2x^\circ = 2\sin x^\circ$ for $0 \leq x \leq 360$ (4)

(b) The diagram below shows parts of the graphs of sine functions f and g . State expressions for $f(x)$ and $g(x)$. (1)

(c) Use your answers to part (a) to find the co-ordinates of A and B. (2)

(d) Hence state the values of x in the interval $0 \leq x \leq 360$ for which $3\sin 2x^\circ < 2\sin x^\circ$. (3)

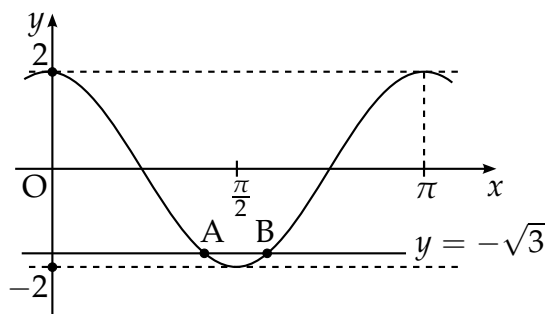


[SQA] 18. The diagram shows the graph of a cosine function from 0 to π .

(a) State the equation of the graph.

(b) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.

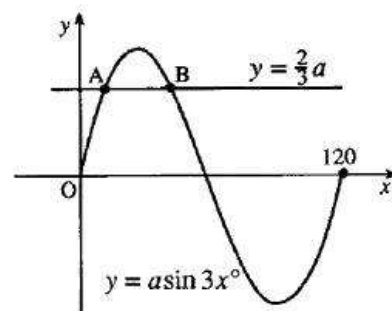
Find the coordinates of B.



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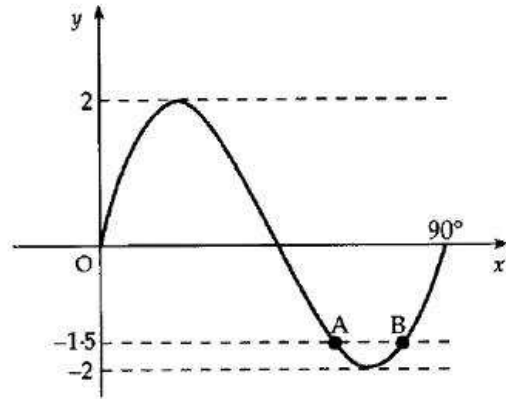
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[SQA] 19. The diagram shows part of the graph of $y = a\sin 3x^\circ$ and the line with equation $y = \frac{2}{3}a$. Find the x -coordinates of A and B.



4

- [SQA] 20. The diagram shows the graph of a sine function from 0° to 90° .
- (a) State the equation of the graph.
- (b) The line with equation $y = -1.5$ intersects the curve at A and B. Find the coordinates of A and B.



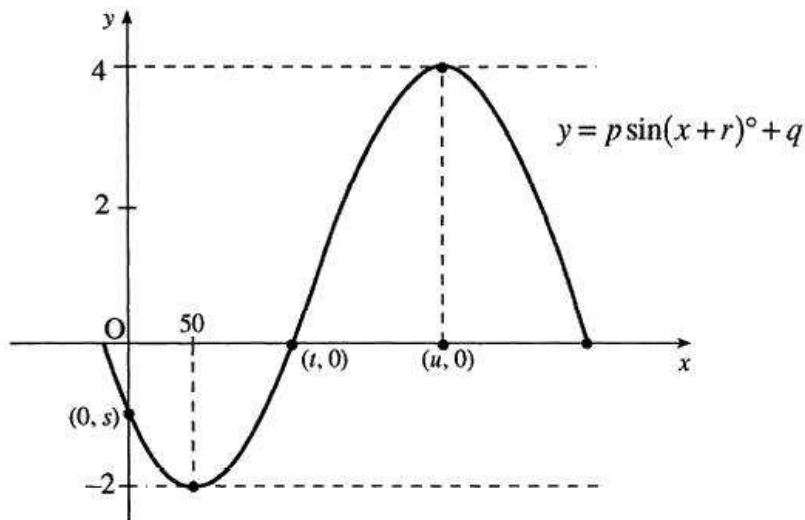
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- [SQA] 21. The sketch represents part of the graph of a trigonometric function of the form $y = p \sin(x+r)^\circ + q$. It crosses the axes at $(0, s)$ and $(t, 0)$, and has turning points at $(50, -2)$ and $(u, 4)$.
- (i) Write down values for p, q, r and u .
- (ii) Find the values for s and t .

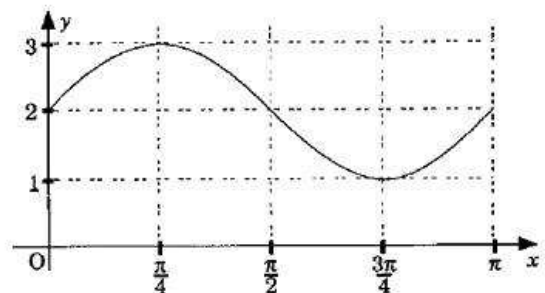
(4)

(4)



- [SQA] 22. The diagram shows the graph of the function $y = a + b \sin cx$ for $0 \leq x \leq \pi$.

- (a) Write down the values of a, b and c .
- (b) Find algebraically the values of x for which $y = 2.5$.

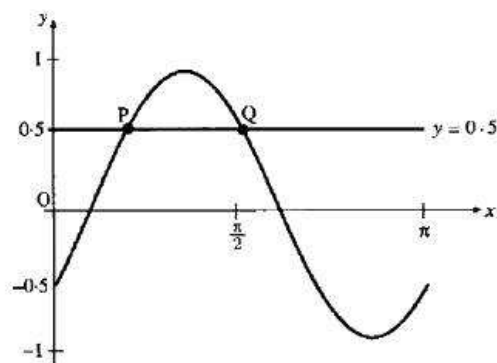


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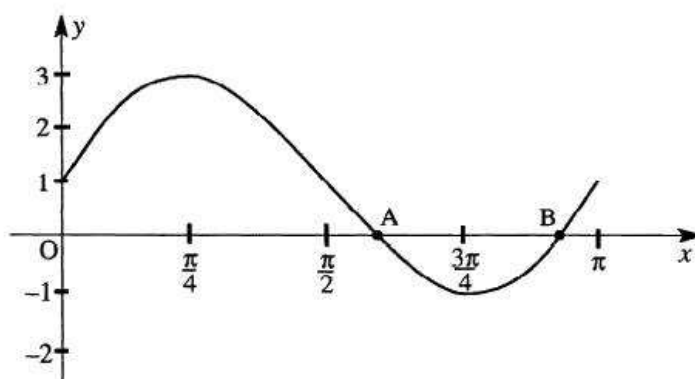
- [SQA] 23. The diagram shows a sketch of the graph of $y = \sin\left(2x - \frac{\pi}{6}\right)$, $0 \leq x \leq \pi$, and the straight line $y = 0.5$. These graphs intersect at P and Q.

Find algebraically the coordinates of P and Q.



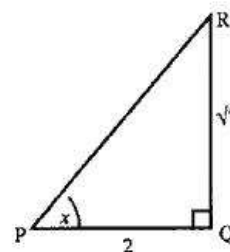
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- [SQA] 24. The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \leq x \leq \pi$.



- (a) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. (5)
- (b) The points $(0, 2)$ and $(\pi, 0)$ are joined by a straight line l . In how many points does l intersect the given graph? (1)
- (c) C is the point on the given graph with an x -coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line l . (3)

- [SQA] 25. Using triangle PQR, as shown, find the exact value of $\cos 2x$.



3

- [SQA] 26. If $\cos \theta = \frac{4}{5}$, $0 \leq \theta < \frac{\pi}{2}$, find the exact value of

(a) $\sin 2\theta$

2

(b) $\sin 4\theta$.

3

[SQA] 27. Given that $\tan \alpha = \frac{\sqrt{11}}{3}$, $0 < \alpha < \frac{\pi}{2}$, find the exact value of $\sin 2\alpha$. 3

[SQA] 28. Given that $\cos D = \frac{2}{\sqrt{5}}$ and $0 < D < \frac{\pi}{2}$, find the exact values of $\sin D$ and $\cos 2D$. 3

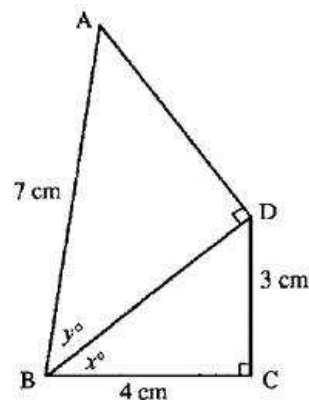
[SQA] 29. Given that $\sin A = \frac{3}{4}$, where $0 < A < \frac{\pi}{2}$, find the exact value of $\sin 2A$. 3

[SQA] 30. For acute angles P and Q , $\sin P = \frac{12}{13}$ and $\sin Q = \frac{3}{5}$.
Show that the exact value of $\sin(P + Q)$ is $\frac{63}{65}$. 3

[SQA] 31. Find the exact value of $\sin \theta^\circ + \sin(\theta^\circ + 120^\circ) + \cos(\theta^\circ + 150^\circ)$. 3

[SQA] 32. If x° is an acute angle such that $\tan x^\circ = \frac{4}{3}$, show that the exact value of $\sin(x^\circ + 30^\circ)$ is $\frac{4\sqrt{3} + 3}{10}$. 3

[SQA] 33. The diagram shows two right-angled triangles ABD and BCD with $AB = 7\text{cm}$, $BC = 4\text{cm}$ and $CD = 3\text{cm}$.
Angle $DBC = x^\circ$ and angle $ABD = y^\circ$.
Show that the exact value of $\cos(x + y)^\circ$ is $\frac{20 - 6\sqrt{6}}{35}$. 3



[SQA] 34. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.

Find the exact value of

(a) $\sin 2A$ 2

(b) $\cos 2A$ 1

(c) $\sin(2A + B)$. 2

[SQA] 35. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(a) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$.

2

(b) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$.

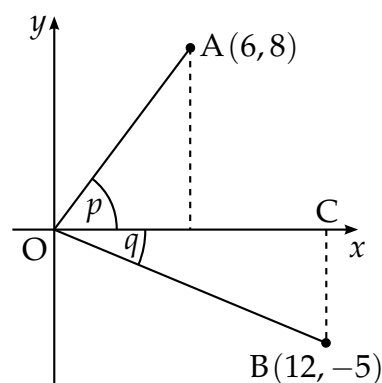
(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$.

5

[SQA] 36. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, -5). Angle $AOC = p$ and angle $COB = q$.

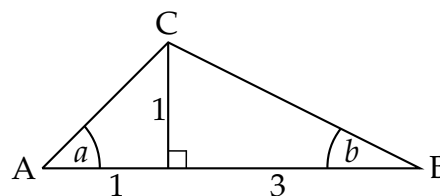
Find the exact value of $\sin(p + q)$.

4



[SQA] 37. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$.

4



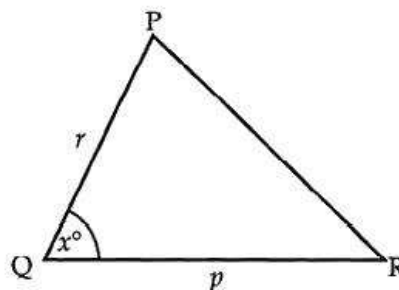
[SQA] 38. The diagram shows an isosceles triangle PQR in which $PR = QR$ and angle $PQR = x^\circ$.

(a) Show that $\frac{\sin x^\circ}{p} = \frac{\sin 2x^\circ}{r}$.

(3)

(b) (i) State the value of x° when $p = r$.

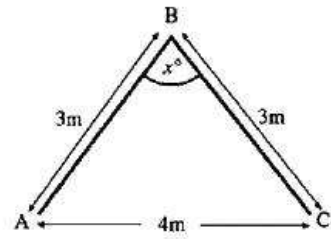
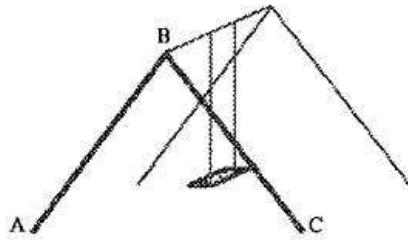
(ii) Using the fact that $p = r$, solve the equation in (a) above, to justify your stated value of x° .



(5)

- [SQA] 39. The framework of a child's swing has dimensions as shown in the diagram on the right.
Find the exact value of $\sin x^\circ$.

5



[END OF WRITTEN QUESTIONS]