Higher

Mathematics

UNIT 1

Specimen NAB Assessment

HSN21510

UNIT 1

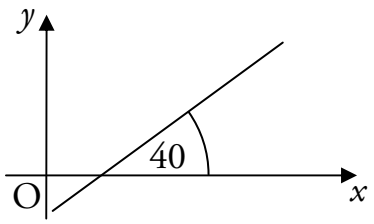
Specimen NAB Assessment

Outcome 1

1. A line passes through the points $A(4,-3)$ and $B(-6,2)$.

   Find the equation of this line. **3**

2. A line makes an angle of 40 with the positive direction of the x-axis, as shown in the diagram.



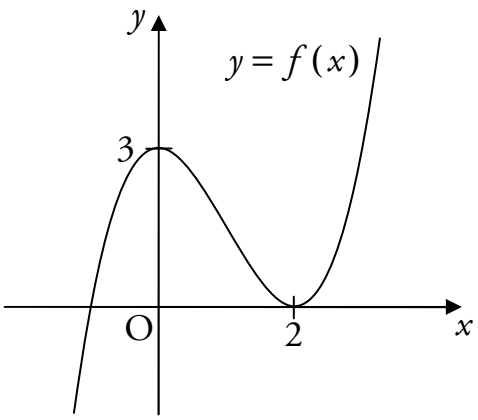
   Find the gradient of this line. **1**

3. (a) Write down the gradient of a line parallel to $y = 4x + 1$. **1**

   (b) Write down the gradient of a line perpendicular to $y = 4x + 1$. **1**

Outcome 2

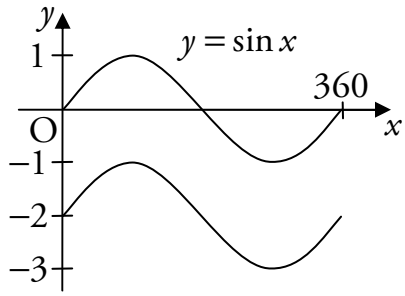
4. The diagram below shows part of the graph of $y = f(x)$.



   (a) Sketch the graph of $y = -f(x)$. **1**

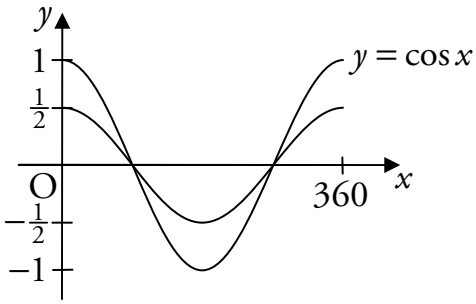
   (b) On a separate diagram, sketch the graph of $y = f(x+4)$. **1**

5. (a) The diagram below shows the curve $y = \sin x$ and a related curve.



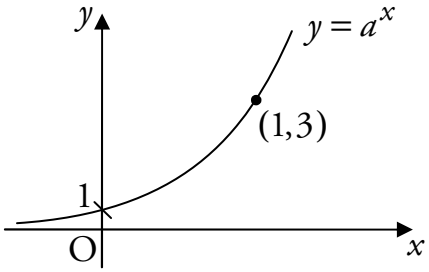
Write down the equation of the related curve. **1**

(b) The diagram below shows the curve $y = \cos x$ and a related curve.



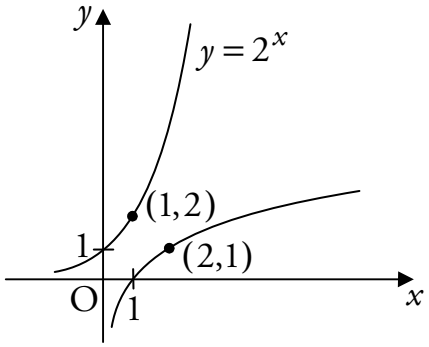
Write down the equation of the related curve. **1**

6. The curve $y = a^x$ is shown in the diagram below.



Given that the curve passes through the point $(1,3)$, write down the value of a. **1**

7. The diagram below shows the graph of the function $f(x) = 2^x$ and its inverse function.



Write down the formula for the inverse function. **1**

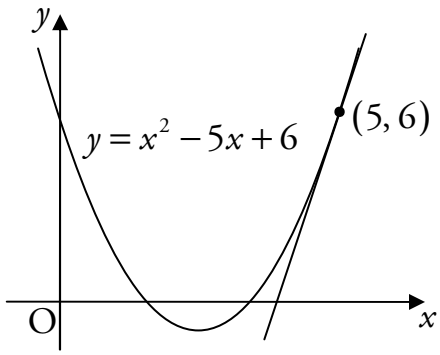
8. (a) Two functions f and g are defined by $f(x)=x^3$ and $g(x)=2x-4$.
Find an expression for $f(g(x))$. **1**

(b) Functions h and k are defined on suitable domains by $h(x)=5x$ and $k(x)=\tan x$.
Find an expression for $k(h(x))$. **1**

Outcome 3

9. Given that $y=\dfrac{x^5-3}{x^3}$ for $x\neq 0$, find $\dfrac{dy}{dx}$. **4**

10. The curve with equation $y=x^2-5x+6$ is shown below.



Find the gradient of the tangent to the curve at the point $(5,6)$. **4**

11. A curve has equation $y=\frac{1}{3}x^3-4x^2+12x-3$.
Find the stationary points on the curve and, using differentiation, determine their nature. **8**

Outcome 4

12. A pond is treated weekly with a chemical to ensure that the number of bacteria is kept low. It is estimated that the chemical kills 68% of all bacteria. Between the weekly treatments, it is estimated that 600 million new bacteria appear. There are u_n million bacteria at the start of a particular week.

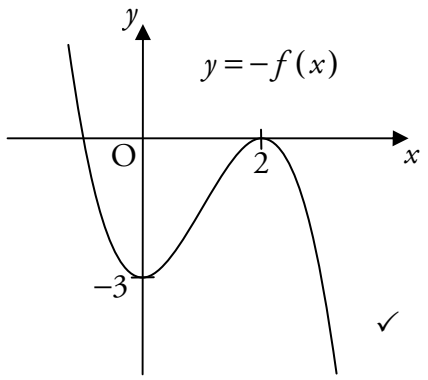
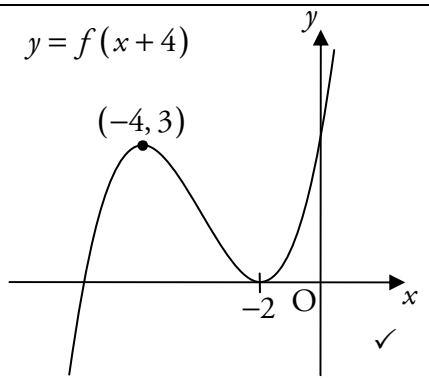
(a) Write down a recurrence relation for u_{n+1}, the number of millions of bacteria at the start of the next week. **1**

(b) Find the limit of the sequence generated by this recurrence relation and explain what the limit means in the context of this question. **3**

Marking Instructions

Pass Marks

Outcome 1	Outcome 2	Outcome 3	Outcome 4
$\frac{4}{6}$	$\frac{6}{8}$	$\frac{11}{16}$	$\frac{3}{4}$

Outcome 1 – Straight Lines	
1. $m = \frac{2-(-3)}{-6-4}$ ✓ $= \frac{5}{-10}$ $= -\frac{1}{2}$ ✓ $y - 2 = -\frac{1}{2}(x+6)$ ✓ $2y - 4 = -x - 6$ $x + 2y + 2 = 0$	• Use gradient formula • Calculate gradient • Equation of line **3**
2. $m = \tan 40$ $= 0{\cdot}84$ (to 2 d.p.) ✓	• Calculate gradient **1**
3. (a) 4 ✓	• State gradient **1**
(b) $-\frac{1}{4}$ ✓	• State gradient **1**
Outcome 2 – Functions and Graphs	
4. (a)  ✓	• Sketch showing images of given points **1**
(b)  ✓	• Sketch showing images of given points **1**

5. (a) $y = \sin x \ -2$ ✓	• Identify equation 1
(b) $y = \frac{1}{2}\cos x$ ✓	• Identify equation 1
6. Since $y = 3$ when $x = 1$: $a^1 = 3$ $a = 3$ ✓	• State the value of a 1
7. $f^{-1}(x) = \log_2 x$ ✓	• State formula for inverse 1
8. (a) $f(g(x)) = f(2x-4)$ $= (2x-4)^3$ ✓	• Expression for composite function 1
(b) $k(h(x)) = k(5x)$ $= \tan 5x$ ✓	• Expression for composite function 1
Outcome 3 – Differentiation	
9. $y = \frac{x^5}{x^3} - \frac{3}{x^3}$ $= x^2$✓$-3x^{-3}$✓ $\frac{dy}{dx} = 2x$✓$+9x^{-4}$✓	• Simplify first term • Simplify second term • Differentiate first term • Differentiate second term 4
10. Gradient of tangent is given by $\frac{dy}{dx}$ ✓ $\frac{dy}{dx} = 2x - 5$ ✓ At $x = 5$✓, $m = 2\times 5 - 5$ $= 5$ ✓	• Know to differentiate • Differentiate • Know to evaluate derivative • Calculate gradient 4

11. $\frac{dy}{dx}\checkmark = x^2 - 8x + 12\checkmark$ Stationary points exist where $\frac{dy}{dx} = 0$ $x^2 - 8x + 12 = 0$ ✓ $(x-6)(x-2) = 0$ $x = 2$ or $x = 6$ ✓ To find y-coordinates: At $x = 6$, $y = \frac{1}{3}(6)^3 - 4(6)^2 + 12(6) - 3$ $= -3$ At $x = 2$, $y = \frac{1}{3}(2)^3 - 4(2)^2 + 12(2) - 3$ $= 7\frac{2}{3}$ ✓ Stationary points are at $\left(2, 7\frac{2}{3}\right)$ and $(6, -3)$ $\begin{array}{c\|ccccc} x & \rightarrow & 2 & \rightarrow & 6 & \rightarrow \\ \hline \frac{dy}{dx} & + & 0 & - & 0 & + \\ \text{sketch} & / & — & \backslash & — & / \end{array}$ ✓ $\left(2, 7\frac{2}{3}\right)$ is a maximum turning point ✓ $(6, -3)$ is a minimum turning point ✓	• Know to differentiate • Differentiate • Set derivative equal to 0 • Find x-coordinates of stationary points • Find y-coordinates of stationary points • Method to determine nature • Nature of one stationary point • Nature of second stationary point **8**
Outcome 4 – Sequences	
12. (a) $u_{n+1} = 0{\cdot}32u_n + 600$	• State recurrence relation **1**
(b) A limit l exists since $-1 < 0{\cdot}32 < 1$ $l = \frac{600}{1 - 0.32}$ ✓ $= 882{\cdot}35$ ✓ (to 2 d.p.) In the long term, the number of bacteria will settle around 882 million ✓	• Know how to calculate limit • Calculate limit • Interpret limit **3**

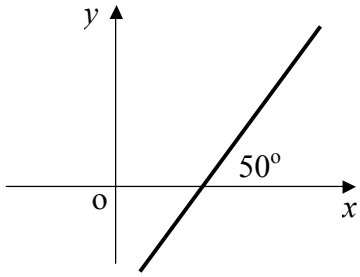
Practice Assessment (1) Unit 1 - Mathematics 1 (H)

Outcome 1 *Marks*

1. A line passes through the points (2,-7) and (6,1).

Find the equation of this line. **(3)**

2. A line makes an angle of 50° with the positive direction of the x-axis, as shown in the diagram, where the scales on the axes are equal.



Find the gradient of the line. **(1)**

3. (a) Write down the **gradient** of any line parallel to $y = \frac{1}{2}x + 3$. **(1)**

(b) Write down the **gradient** of a line perpendicular to $y = -3x - 1$. **(1)**

Outcome 2

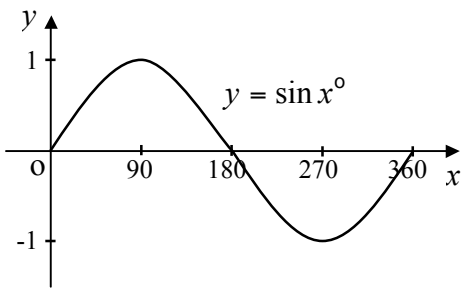
4. **See worksheet**.

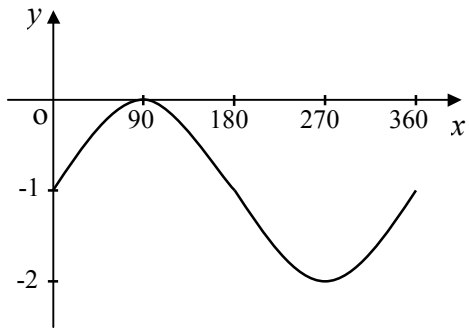
Diagrams 1 and 2 on the worksheet show part of the graph of $y = f(x)$.

(a) On Diagram 1, draw the graph of $y = -f(x)$. **(1)**

(b) On Diagram 2, draw the graph of $y = f(x + 4)$. **(1)**

5. (a) The diagrams below show part of the graph of $y = \sin x^{\circ}$ and the graph of a related function. Write down the equation of the related function.

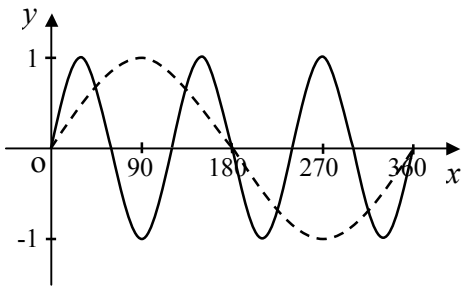




(1)

(b) The diagram shows part of the graph of $y = \sin x^{\circ}$ (drawn as a broken line) and the graph of a related function.

Write down the equation of the related graph.



(1)

6. **See worksheet**.

The graph of $y = 2^x$ is shown in Diagram 3 on the worksheet.

Write down the equation of the graph of the exponential function of the form $y = a^x$ which passes through the point (2,9) as shown on the worksheet. **(1)**

7. **See worksheet**.

Diagram 4 on the worksheet shows part of the graph of the function $y = 6^x$ and its inverse function.

Write down the equation of the inverse function. **(1)**

8. (a) Two functions *f* and *g* are given by $f(x) = x^2 - 1$ and $g(x) = 3x - 1$.

Obtain an expression for $f(g(x))$. **(1)**

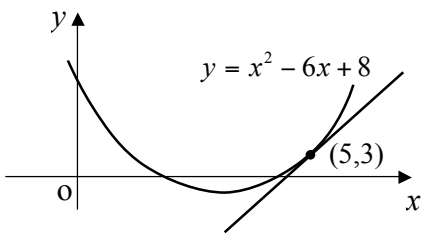
(b) Functions *h* and *k*, defined on suitable domains, are given by $h(x) = 4x$ and $k(x) = \cos x$. Find $k(h(x))$. **(1)**

Outcome 3

9. Given $y = \dfrac{1 + x^4}{x^2}$, find $\dfrac{dy}{dx}$. **(4)**

10. The diagram shows a sketch of the curve with equation $y = x^2 - 6x + 8$ with a tangent drawn at the point (5,3).

Find the gradient of this tangent. **(4)**



11. Find the coordinates of the stationary points of the curve with equation $y = \frac{1}{3}x^3 - x + 1$.

Using differentiation determine their nature. **(8)**

Outcome 4

12. In a small colony 20% of the existing insects are eaten by predators each day, however during the night 400 insects are hatched. There are U_n insects at the start of a particular day.

(a) Write down a recurrence relation for U_{n+1}, the number of insects at the start of the next day. **(1)**

(b) Find the limit of the sequence generated by this recurrence relation and explain what the limit means in the context of this question. **(3)**

Unit 1 - Practice Assessment (1) **Worksheet for Questions 4, 6 and 7**

Name : _____ Class : _____

Question 4

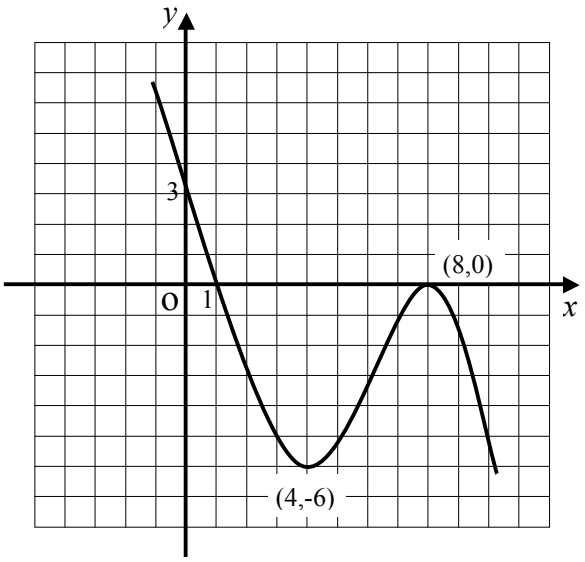


Diagram 1

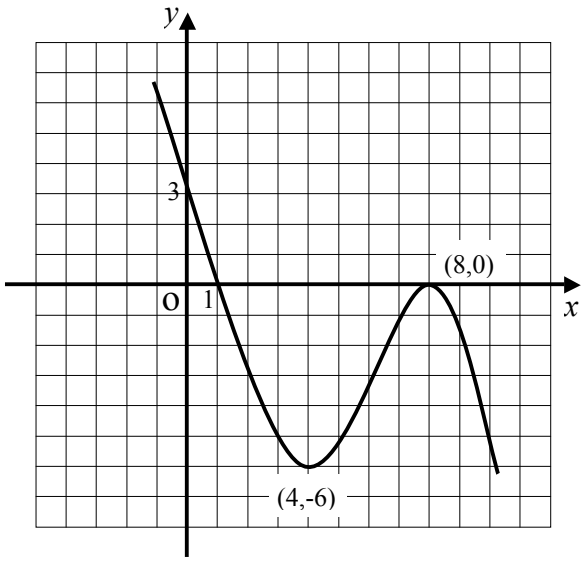


Diagram 2

Question 6

The equation of the graph passing through (2,9) is .....

$y =$ .....

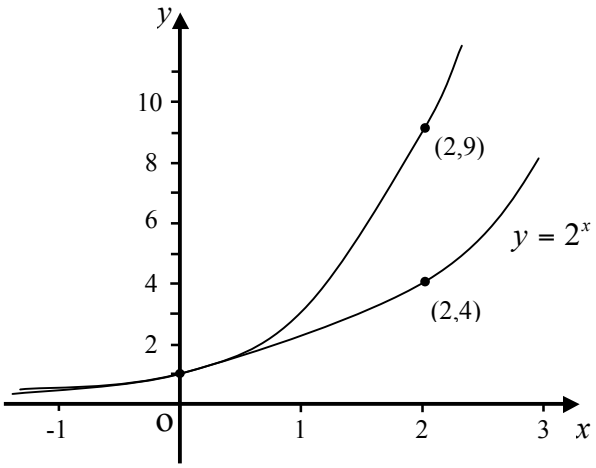


Diagram 3

Question 7

The equation of the graph passing through (1,0) is

.....

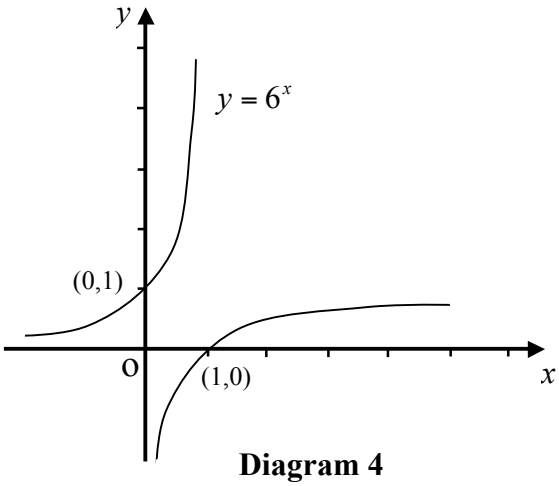


Diagram 4

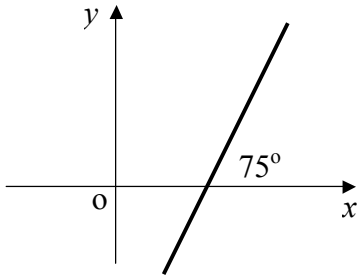
Practice Assessment (2) Unit 1 - Mathematics 1 (H)

Outcome 1 *Marks*

1. A line passes through the points (-1,3) and (-4,2).

Find the equation of this line. **(3)**

2. A line makes an angle of 75° with the positive direction of the x-axis, as shown in the diagram, where the scales on the axes are equal.



Find the gradient of the line. **(1)**

3. (a) Write down the **gradient** of any line parallel to $y = -x - 2$. **(1)**

(b) Write down the **gradient** of a line perpendicular to $y = \frac{3}{2}x - 1$. **(1)**

Outcome 2

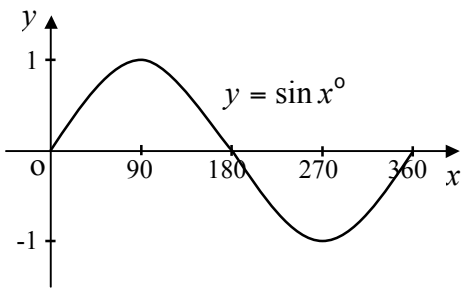
4. **See worksheet**.

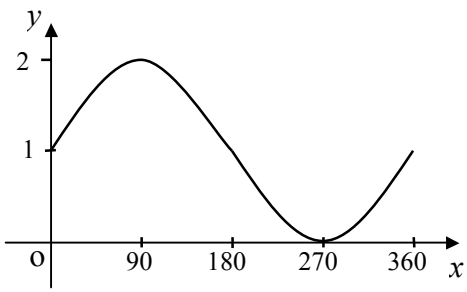
Diagrams 1 and 2 on the worksheet show part of the graph of $y = f(x)$.

(a) On Diagram 1, draw the graph of $y = -f(x)$. **(1)**

(b) On Diagram 2, draw the graph of $y = f(x-4)$. **(1)**

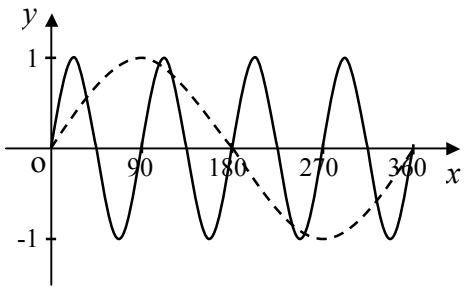
5. (a) The diagrams below show part of the graph of $y = \sin x^{\circ}$ and the graph of a related function. Write down the equation of the related function.





(1)

(b) The diagram shows part of the graph of $y = \sin x^{\circ}$ (drawn as a broken line) and the graph of a related function.



Write down the equation of the related graph. **(1)**

6. **See worksheet**.

The graph of $y = 3^x$ is shown in Diagram 3 on the worksheet.

Write down the equation of the graph of the exponential function of the form $y = a^x$ which passes through the point (1,6) as shown on the worksheet. **(1)**

7. **See worksheet**.

Diagram 4 on the worksheet shows part of the graph of the function $y = 7^x$ and its inverse function.

Write down the equation of the inverse function. **(1)**

8. (a) Two functions *f* and *g* are given by $f(x) = 2x^2$ and $g(x) = x + 1$.

Obtain an expression for $f(g(x))$. **(1)**

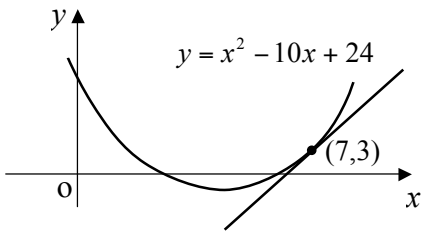
(b) Functions *h* and *k*, defined on suitable domains, are given by $h(x) = \sin x$ and $k(x) = \frac{1}{2}x$. Find $k(h(x))$. **(1)**

Outcome 3

9. Given $y = \dfrac{x^4 + 2}{x^3}$, find $\dfrac{dy}{dx}$. **(4)**

10. The diagram shows a sketch of the curve with equation $y = x^2 - 10x + 24$ with a tangent drawn at the point (7,3).

Find the gradient of this tangent. **(4)**



11. Find the coordinates of the stationary points of the curve with equation $y = \frac{2}{3}x^3 + x^2 - 4x$.

Using differentiation determine their nature. **(8)**

Outcome 4

12. In a small rabbit colony one eighth of the existing rabbits are eaten by predators each summer, however during the winter 24 rabbits are born.
There are U_n rabbits at the start of a particular summer.

(a) Write down a recurrence relation for U_{n+1}, the number of rabbits at the start of the next summer. **(1)**

(b) Find the limit of the sequence generated by this recurrence relation and explain what the limit means in the context of this question. **(3)**

Unit 1 - Practice Assessment (2) **Worksheet for Questions 4, 6 and 7**

Name : _____ Class : _____

Question 4

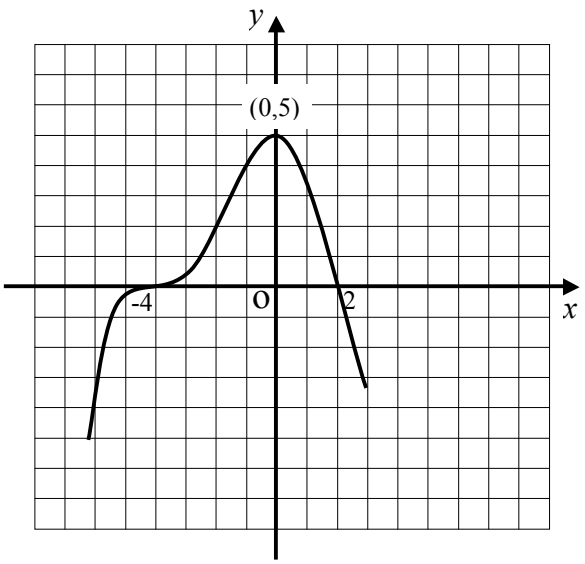


Diagram 1

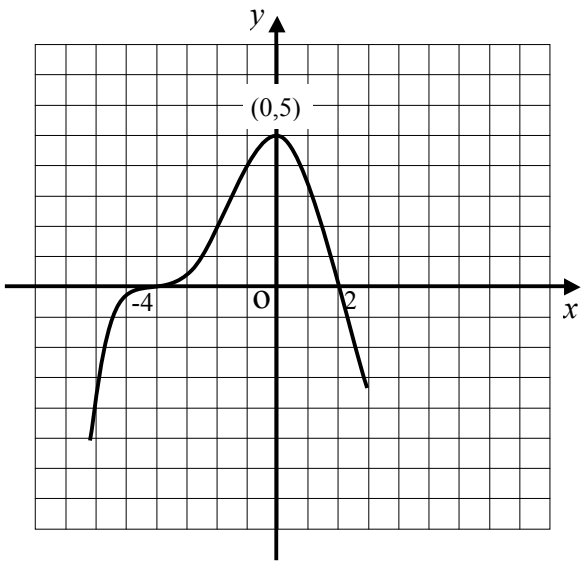


Diagram 2

Question 6

The equation of the graph passing through (1,6) is .....

$y =$ .....

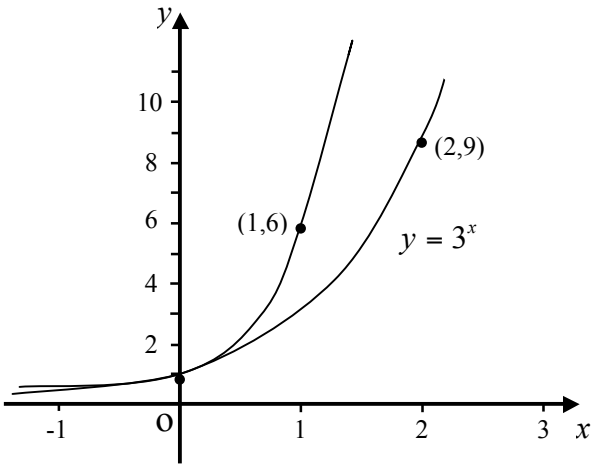


Diagram 3

Question 7

The equation of the graph passing through (1,0) is

.....

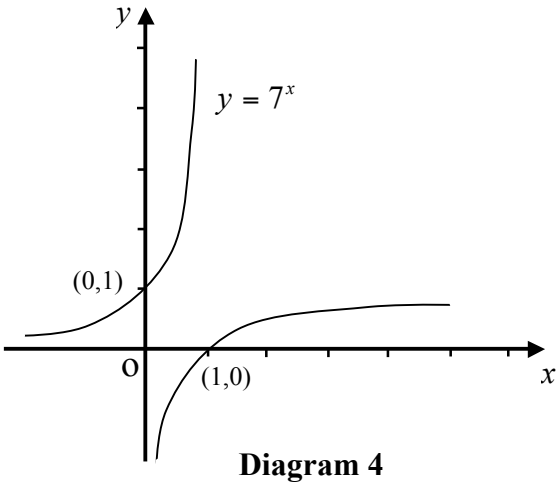


Diagram 4

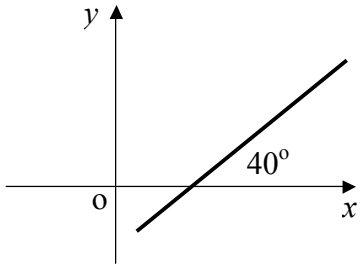
Practice Assessment (3) Unit 1 - Mathematics 1 (H)

Outcome 1 *Marks*

1. A line passes through the points (4,-4) and (2,6).

Find the equation of this line. **(3)**

2. A line makes an angle of 40° with the positive direction of the x-axis, as shown in the diagram, where the scales on the axes are equal.



Find the gradient of the line. **(1)**

3. (a) Write down the **gradient** of any line parallel to $y = -\frac{3}{4}x + 9$. **(1)**

(b) Write down the **gradient** of a line perpendicular to $y = -x - 2$. **(1)**

Outcome 2

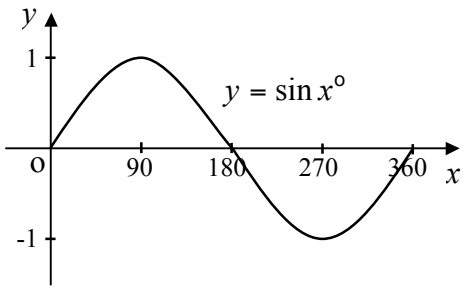
4. **See worksheet**.

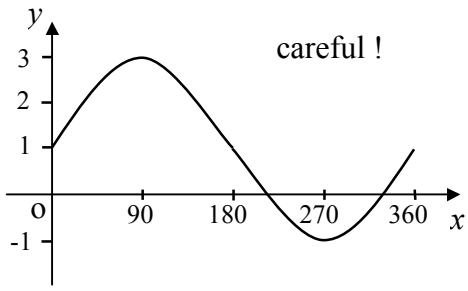
Diagrams 1 and 2 on the worksheet show part of the graph of $y = f(x)$.

(a) On Diagram 1, draw the graph of $y = -f(x) + 2$. **(1)**

(b) On Diagram 2, draw the graph of $y = f(x - 3)$. **(1)**

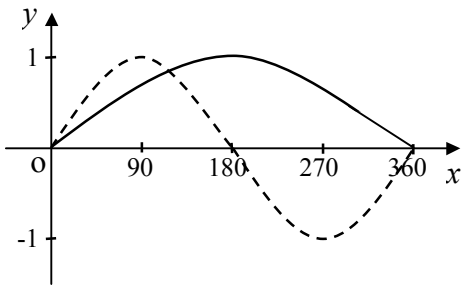
5. (a) The diagrams below show part of the graph of $y = \sin x^{\circ}$ and the graph of a related function. Write down the equation of the related function.





(1)

(b) The diagram shows part of the graph of $y = \sin x^{\circ}$ (drawn as a broken line) and the graph of a related function.



Write down the equation of the related graph. **(1)**

6. **See worksheet**.

The graph of $y = 5^x$ is shown in Diagram 3 on the worksheet.

Write down the equation of the graph of the exponential function of the form $y = a^x$ which passes through the point (1,2) as shown on the worksheet. **(1)**

7. **See worksheet**.

Diagram 4 on the worksheet shows part of the graph of the function $y = 3^x$ and its inverse function.

Write down the equation of the inverse function. **(1)**

8. (a) Two functions *f* and *g* are given by $f(x) = x^2 + x$ and $g(x) = 3x + 1$.

Obtain an expression for $f(g(x))$. **(1)**

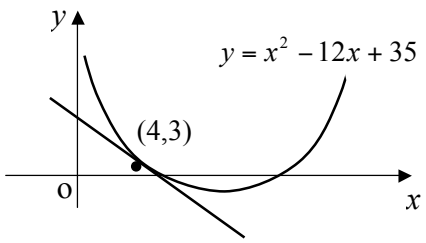
(b) Functions *k* and *h*, defined on suitable domains, are given by $k(x) = \cos x$ and $h(x) = (2x + \pi)$. Find $k(h(x))$. **(1)**

Outcome 3

9. Given $y = \dfrac{3 + x^6}{x^4}$, find $\dfrac{dy}{dx}$. **(4)**

10. The diagram shows a sketch of the curve with equation $y = x^2 - 12x + 35$ with a tangent drawn at the point (4,3).

Find the gradient of this tangent. **(4)**



11. Find the coordinates of the stationary points of the curve with equation $y = x^3 - 3x^2 - 9x + 15$.

Using differentiation determine their nature. **(8)**

Outcome 4

12. For an established ant hill 6% of the worker ants fail to return at the end of each day. However, during the night 540 worker ants are hatched.
There are U_n worker ants at the start of a particular day.

(a) Write down a recurrence relation for U_{n+1}, the number of worker ants at the start of the next day. **(1)**

(b) Find the limit of the sequence generated by this recurrence relation and explain what the limit means in the context of this question. **(3)**

Unit 1 - Practice Assessment (3) **Worksheet for Questions 4, 6 and 7**

Name : _____ Class : _____

Question 4

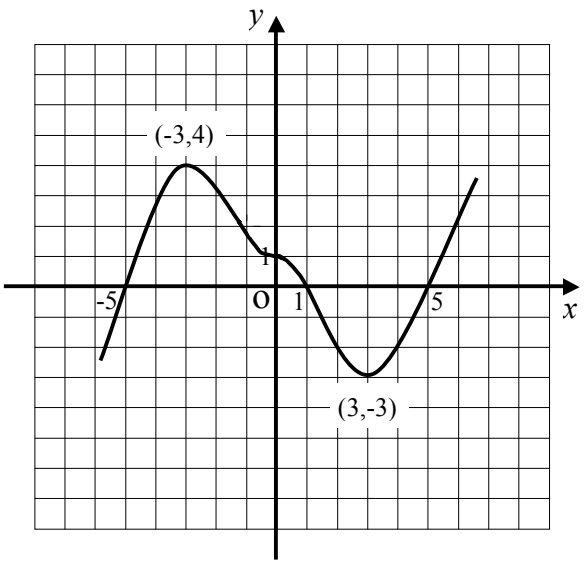


Diagram 1

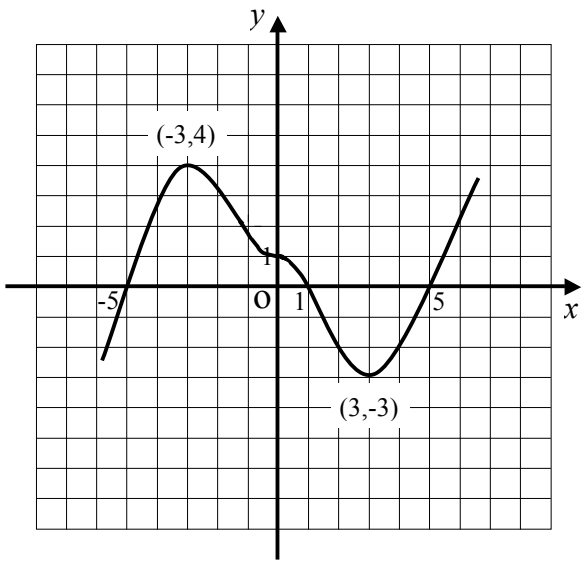


Diagram 2

Question 6

The equation of the graph passing through (1,2) is .....

$y =$ .....

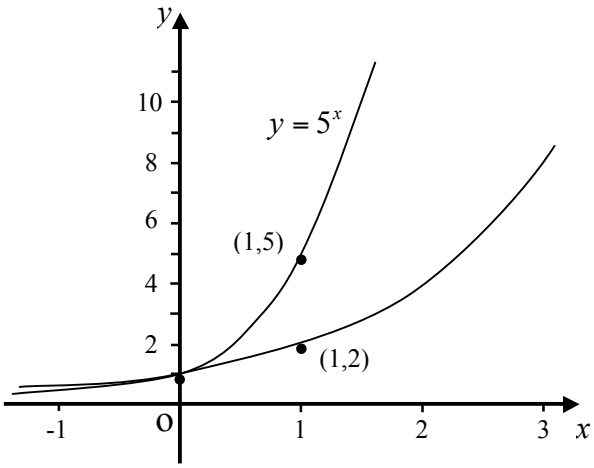


Diagram 3

Question 7

The equation of the graph passing through (1,0) is

.....

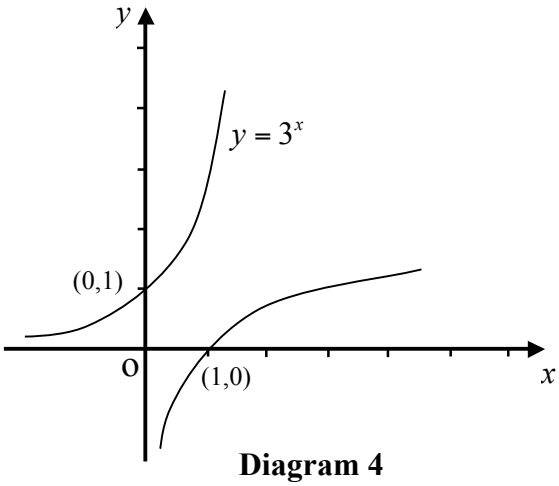


Diagram 4

Unit 1 - Practice Assessments **Answers**

Practice Assessment 1

Outcome 1 : **1.** $m = 2$, $y = 2x - 11$ ($y - 1 = 2(x - 6)$ *or* $y + 7 = 2(x - 2)$)

2. $m = 1 \cdot 19$ **3.** (a) $m = \frac{1}{2}$ (b) $m = \frac{1}{3}$

Outcome 2 : **4.** (a) diagram (reflection in x-axis) (b) diagram (translated 4 units left)

5. (a) $y = \sin x^{\circ} - 1$ (b) $y = \sin 3x^{\circ}$ **6.** $y = 3^x$ **7.** $y = \log_6 x$

8. (a) $f(g(x)) = (3x - 1)^2 - 1 \Rightarrow 9x^2 - 6x$ (b) $k(h(x)) = \cos 4x$

Outcome 3 : **9.** $\dfrac{dy}{dx} = -2x^{-3} + 2x$ **10.** $m = 4$ **11.** $(-1, \frac{5}{3})$, max , $(1, \frac{1}{3})$, min

Outcome 4 : **12.** (a) $U_{n+1} = 0 \cdot 8U_n + 400$ (b) $L = 2000$, + explanation

Practice Assessment 2

Outcome 1 : **1.** $m = \frac{1}{3}$, $3y = x + 10$ ($y - 3 = \frac{1}{3}(x + 1)$ *or* $y - 2 = \frac{1}{3}(x + 4)$)

2. $m = 3 \cdot 73$ **3.** (a) $m = -1$ (b) $m = -\frac{2}{3}$

Outcome 2 : **4.** (a) diagram (reflection in x-axis) (b) diagram (translated 4 units right)

5. (a) $y = \sin x^{\circ} + 1$ (b) $y = \sin 4x^{\circ}$ **6.** $y = 6^x$ **7.** $y = \log_7 x$

8. (a) $f(g(x)) = 2(x + 1)^2 \Rightarrow 2x^2 + 4x + 2$ (b) $k(h(x)) = \frac{1}{2}\sin x$

Outcome 3 : **9.** $\dfrac{dy}{dx} = 1 - 6x^{-4}$ **10.** $m = 4$ **11.** $(-2, 6\frac{2}{3})$, max , $(1, -2\frac{1}{3})$, min

Outcome 4 : **12.** (a) $U_{n+1} = \frac{7}{8}U_n + 24$ (b) $L = 192$, + explanation

Practice Assessment 3

Outcome 1 : **1.** $m = -5$, $y = -5x + 16$ ($y + 4 = -5(x - 4)$ *or* $y - 6 = -5(x - 2)$)

2. $m = 0 \cdot 84$ **3.** (a) $m = -\frac{3}{4}$ (b) $m = 1$

Outcome 2 : **4.** (a) diagram (reflection in x-axis then up 2) (b) diagram (translated 3 units right)

5. (a) $y = 2\sin x^{\circ} + 1$ (b) $y = \sin \frac{1}{2}x^{\circ}$ **6.** $y = 2^x$ **7.** $y = \log_3 x$

8. (a) $f(g(x)) = (3x + 1)^2 + 3x + 1 \Rightarrow 9x^2 + 9x + 2$ (b) $k(h(x)) = \cos(2x + \pi)$

Outcome 3 : **9.** $\dfrac{dy}{dx} = -12x^{-5} + 2x$ **10.** $m = -4$ **11.** $(-1, 20)$, max , $(3, -12)$, min

Outcome 4 : **12.** (a) $U_{n+1} = 0 \cdot 94U_n + 540$ (b) $L = 9000$, + explanation