Help Your Child With Higher Maths

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Introduction

We've designed this booklet so that you can use it with your child throughout the session, as he/she moves through the Higher course, in order to help them remember key facts and methods. There are separate sections covering the three units of the Higher, as well as one on Credit/Intermediate 2 revision. Obviously there's no point trying to revise Unit 3 work when your child hasn't yet covered it in class, so it's worth giving a rough timetable for the course:

Unit 1 is typically completed by mid-November
Unit 2 is typically completed by February
Unit 3 is typically completed by April
(The section on Credit/Intermediate 2 revision is fair game at any time of the course – your child should already know it all!)

The booklet is **not** an exhaustive summary of the content of the Higher – a separate booklet is available for that, should you wish to read it! – nor is it meant to replace the set of much more detailed Higher notes which your child should be building up over the session. But the booklet does summarise the key facts and methods which your child will need to be familiar with, if they are to have a chance of passing the Higher. Regular revision of these facts and methods will pay real dividends: it is much better to continually revise throughout the year, than to attempt to "cram" it all in at the last minute.

How to use the booklet

Your child could use this booklet on their own, but we think it would be much better if they had someone else to "test" them on the content. Basically, all you have to do is read out the question on the left-hand column of the page, and all they have to do is give the correct answer (more or less) which is shown on the right. If you prefer, you could simply show your child the question, covering up the answer as you do so.

So by the end of Unit 1 (mid-November), for example, you could test your child on the entire Unit 1 content. However we think it's much better to revise more regularly than that, so we'd suggest that you find out from your child which section they are on (or have already completed) and test them on that, at reasonably regular intervals. If you can do this then you will be making a real contribution to your child's knowledge of the course. In a sense, you will be helping them to remember their "lines" for the performance that will be the Higher Mathematics examination.

If you lack confidence in maths yourself

Don't worry! We have tried to explain the terminology and notation as we go along, but if you are in doubt, then either simply show the question to your child instead or ask them to explain to you how to say it. You may find that some of your child's answers differ slightly from what is here: this may not necessarily mean that they are wrong, as different teachers will naturally teach things in slightly different ways. If in doubt, check with your child. If **still** in doubt, please feel free to get in touch with us at the school.

Thanks in advance for your help, and good luck!

The Straight Line	
What is the gradient of a horizontal line? **What is the equation of a horizontal line?** **So the equation of the x-axis is…?**	$m = 0$ $y = b$ $y = 0$
What is the gradient of a vertical line? **What is the equation of a vertical line?** **So the equation of the y-axis is…?**	m is undefined $x = a$ $x = 0$
What is the gradient formula?	$m = \dfrac{y_2 - y_1}{x_2 - x_1}$
How do you find the equation of a straight line?	You need to know the gradient of the line and a point on the line, then you use $y - b = m(x - a)$
How do you find the gradient of a straight line if you know its equation?	Rearrange to the form $y = mx + c$
How do you find the size of the angle between a line and the x-axis?	Use $m = \tan\theta$
What is the rule for parallel lines?	$m_1 = m_2$
What is the rule for perpendicular lines?	$m_1 \times m_2 = -1$
How do you find where two lines meet?	Use simultaneous equations or use $y = y$
What does it mean if three points are said to be collinear?	They lie in a straight line
How do you show that three points A, B and C *are* collinear?	Show that $m_{AB} = m_{BC}$ so the lines are parallel, but B is a common point so A, B and C are collinear
SPECIAL LINES **What is a perpendicular bisector?** **What is a median of a triangle?** **What is an altitude of a triangle?**	A line which bisects (cuts in half) a given line at right-angles – find midpoint and use $m_1 \times m_2 = -1$ A line drawn from one vertex to the midpoint of the opposite side – find midpoint then gradient A line drawn from one vertex to the opposite side, meeting it at right-angles – use $m_1 \times m_2 = -1$
What is the distance formula?	$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ or use Pythagoras

Differentiation	
How do you differentiate?	Multiply by the power, then decrease the power by one
How do you prepare for differentiation?	Change any roots into powers x must not be on the denominator (bottom) of any fraction Any pairs of brackets should be expanded
What notation or phrases can we also use to represent differentiation?	$f'(x)$ (f "dashed" of x), $\frac{dy}{dx}$ (dy "by" dx) Rate of change, Gradient function, Derived function
How do we find the gradient of the tangent to a curve at a given value of x? **How do we find the equation of the tangent to a curve?**	Differentiate, then sub x in to find the gradient As above, but we also need to find the y co-ordinate by substituting x into the original expression, then use $y-b=m(x-a)$
A function is increasing when…? **And decreasing when…?** **And stationary when…?**	$f'(x)>0$ $f'(x)<0$ $f'(x)=0$
How do you find the stationary points of a function?	At the stationary points $f'(x)=0$ Differentiate then solve to find the x values Substitute back in to find y values Use a nature table to determine the nature
How do you find where a function is increasing or decreasing?	Differentiate then use a nature table (or solve the inequality)
How do you show that a function is ALWAYS increasing (decreasing)?	Differentiate then complete the square to show that $f'(x)>0$ ($f'(x)<0$) for all values of x
How do you find the solution to an optimisation problem?	Investigate stationary points (and end-points if necessary)
What do you get if you differentiate distance? **What do you get if you differentiate speed?**	Speed Acceleration

Families of graphs	
Given the graph of $y=f(x)$, how do you	

sketch… $y = f(x) + k$ $y = f(x) - k$ $y = kf(x)$ $y = -f(x)$	Move graph **up** k units Move graph **down** k units Stretch graph up/down by factor of k Reflect graph in the x-axis
Given the graph of $y = f(x)$, how do you sketch… $y = f(x+k)$ $y = f(x-k)$ $y = f(kx)$ $y = f(-x)$	Move graph k units to the **left** Move graph k units to the **right** Compress the graph by a factor of k horizontally Reflect graph in the y-axis
Given the graph of $y = f(x)$, how do you sketch… $y = f'(x)$ (the **derived** graph)	The x-coordinates of the stationary points become zeroes of the graph; then look at the gradient of the curve between these points to decide on shape (positive – above y-axis; negative – below y-axis)

Functions	
What is the domain of a function?	The set of numbers which go INTO the function
What is the range of a function?	The set of numbers which come OUT of the function
How do you find where a function is undefined?	Look for values of x which make the denominator of the fraction equal to zero **or** which lead to negative square roots
How do you find a suitable domain for a function?	Write down an expression for all the values of x for which the function is NOT undefined
If $f(g(x)) = x$, what is the connection between functions f and g? (“f of g of x”)	They are inverses of each other

Trigonometry: graphs and equations

For a trig graph of the form
$y = a\sin bx + c$ **or** $y = a\cos bx + c$,
how do we work out the value of…

a **?**

a tells us the amplitude (the difference between the maximum and minimum values)

b **?**

b tells us how often the graph repeats in $360°$ (so the period of the graph is $360° \div b$)

c **?**

c tells us how far the graph has been moved up or down from its usual starting position

Exact values:
The results in the table opposite should be known – either memorise these, or know how to derive them by drawing triangles.

(eg question: what is $\sin 30°$ **? Answer: ½)**

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	-

(we say $\tan 90°$ is *undefined*)

Radian measure:
You need to be able to convert from degrees to radians and vice-versa.

Some common examples are given opposite and should be memorised: practice asking these both ways.

$180° = \pi$ (pi radians)

$360° = 2\pi$

$0° = 0$

$90° = \frac{\pi}{2}$

$30° = \frac{\pi}{6}$

$45° = \frac{\pi}{4}$

$60° = \frac{\pi}{3}$

What does $\sin^2 x + \cos^2 x$ **always equal?**
(sine squared x plus cos squared x)

1

What does $\frac{\sin x}{\cos x}$ **always equal?**

$\tan x$

Recurrence relations	
What is meant if a recurrence relation is said to be… **Convergent?** **Divergent?**	 There is a limit There is no limit
What is the condition for a recurrence relation to have a limit?	$-1 < a < 1$ (a is between -1 and 1)
How do you find the limit of a recurrence relation?	Use the formula $L = \frac{b}{1-a}$ (or replace u_{n+1} and u_n by L in the original expression and solve)
Given three consecutive terms in a recurrence relation, how can you work out the formula?	Set up two equations using pairs of values then solve simultaneously

Quadratic Functions	
How do you sketch a quadratic curve (parabola)?	1. Find the shape – "happy" or "sad"? 2. Find the roots (if they exist) – ie. where the curve cuts the x-axis (solve $y = 0$) 3. Find where the curve cuts the y-axis ($x = 0$) 4. Use symmetry to find the turning point (or use differentiation)
Completing the square: **Why do we complete the square?**	To allow us to make a quick sketch of the parabola, which allows us to find the turning point
What is the process for completing the square?	Identify the x-coefficient Halve it Square it Add it on/take it away OR: expand brackets and equate coefficients
What form must the expression be in before you can complete the square?	Must be $x^2 + ...$ and not $2x^2$ etc, so take out a common factor if you have to
The discriminant: **What is the condition for…**	
… equal roots?	$b^2 - 4ac = 0$
… two distinct real roots?	$b^2 - 4ac > 0$
… real roots?	$b^2 - 4ac \geq 0$
… non-real roots? (or no real roots)	$b^2 - 4ac < 0$
How do you show that a line is a tangent to a curve?	Substitute the line into the curve and solve the equation to show that there are equal roots (or show that $b^2 - 4ac = 0$)
What does it mean to say that a quadratic is irreducible?	It cannot be factorised

Polynomials	
How do you show that $x-a$ is a factor of $f(x)$?	Use synthetic division (with a) to show that the remainder is zero, or show that $f(a)=0$
How do you factorise a cubic?	First find a linear factor, using synthetic division, then factorise the quadratic from the bottom row of the table.
How do you sketch the graph of a polynomial?	1. Find where the curve crosses the x-axis ($y=0$) and the y-axis ($x=0$) 2. Differentiate and solve $\frac{dy}{dx}=0$ to find the stationary points 3. Use a nature table to determine nature 4. Sketch the graph

Integration	
How do you integrate?	Increase the power by one, then divide by the new power
How do you prepare for integration?	Change any roots into powers x must not be on the denominator (bottom) of any fraction Any pairs of brackets should be expanded
When integrating an indefinite integral (one with no limits), what must we always remember?	+C
Why do we integrate?	To find the area under a curve, or to recover $f(x)$ from $f'(x)$
What do we have to remember when the enclosed area is below the x-axis?	The answer will be negative, so we explain this fact and change the answer to positive
What do we have to remember when the area is partly above and partly below the x-axis?	We have to work out the areas separately (one above x-axis, one below) then add
How do we find the area between two curves or a line and a curve? **How do we find where the curves meet?**	$\int (curve\ above - curve\ below)\,dx$ Use $y = y$ and solve
What do we get if we integrate acceleration? **What do we get if we integrate speed?**	Speed Distance

Compound Angle Formulae	
cos(A + B) = ? **cos(A – B) = ?** **sin(A + B) = ?** **sin(A – B) = ?**	cosAcosB – sinAsinB cosAcosB + sinAsinB sinAcosB + cosAsinB sinAcosB – cosAsinB
When asked to find the exact value of sin, cos or tan, what should you look for? **If a right-angled triangle is not involved, what should you do?**	Right-angled triangles Try to make an expression up which involves right-angled triangles and exact values you know (eg 30, 45, 60 degrees)
If you are given sin, cos or tan and told that the angle is acute ($0° < x° < 90°$, $0 < x < \frac{\pi}{2}$), how can you find the other ratios as exact values?	Draw a right-angled triangle, use Pythagoras to find the missing side, then use SOHCAHTOA
sin2A = ? **cos2A = ? (three possible answers)**	$2\sin A\cos A$ $\cos^2 A - \sin^2 A$ $2\cos^2 A - 1$ $1 - 2\sin^2 A$
How can you expand cos4A, sin4A etc? **How can you expand cos3A etc?**	Write as (2A+2A) then expand using the formulae Write as (2A+A) then expand using the formulae
When solving a trig equation, what two-step process should you follow?	Is it a "straight-forward solve"? If not, then "double angle solve"
How do you recognise and solve a "straight-forward solve"?	sin, cos or tan appears once only Solve to find acute angle, then use ASTC
How do you recognise and solve a "double-angle solve"?	Look for a double angle and a single angle (eg 2A and A) Replace the double-angle formula with an appropriate single expression, then make one side zero and factorise in order to solve
What should you always check at the end of a trig question?	Should the answer be given in degrees or radians?

The Circle	
What kind of circle has equation… $x^2 + y^2 = r^2$ **?** $(x-a)^2 + (y-b)^2 = r^2$ **?** $x^2 + y^2 + 2gx + 2fy + c = 0$ **?**	 centre (0, 0), radius r centre (a,b), radius r centre $(-g,-f)$, radius $\sqrt{g^2 + f^2 - c}$
How do you find the equation of a circle? **Do you need to expand the brackets and tidy up your answer?**	Find the centre and radius, then use $(x-a)^2 + (y-b)^2 = r^2$ No!!
How can you show that an equation does NOT represent a circle?	Try to find the radius – you should be left with the square root of a negative number, which is impossible, or zero
How do you find where a line meets a circle?	Rearrange the line into the form $y =$ or $x =$ (whichever is easier) then substitute this into the circle and solve
How do you show that a line is a tangent to a circle?	As above – you should find equal roots, ie only one point of contact (alternatively, show that $b^2 - 4ac = 0$)
How do you show that a line does not meet a circle at all	As above – this time show that there are no real roots, ie $b^2 - 4ac < 0$
How do you find the equation of a tangent to a circle?	Find the gradient of the radius Use $m_1 \times m_2 = -1$ to find the gradient of the tangent Then use $y - b = m(x - a)$
What is a common tangent?	A line which is a tangent to two circles
How do I show that two circles touch externally?	Show that the distance between the two centres is equal to the sum of the two radii
What is meant by congruent circles? **What is meant by concentric circles?**	Circles that are the same size Circles with the same centre

Vectors	
What is the difference between a vector and a scalar?	A vector has magnitude (size) and direction, whereas a scalar only has magnitude
What is meant by giving a vector in component form? **How would you write this in i, j, k form?**	Writing the answer as a column vector with brackets, eg $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ $a\underline{i} + b\underline{j} + c\underline{k}$
How do you find the magnitude (length) of vector $\underline{u} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ **?**	$\lvert\underline{u}\rvert = \sqrt{a^2 + b^2 + c^2}$
How do you find vector $\overrightarrow{AB}$ **?**	$\overrightarrow{AB} = \underline{b} - \underline{a}$
How do you show that two vectors are parallel?	Show that one vector is a multiple of the other
If point P divides AB in the ratio m:n, how do you find the coordinates of P?	Use the Section Formula: $\underline{p} = \dfrac{1}{m+n}(n\underline{a} + m\underline{b})$ and then write out the coordinates of P (or use ratios to create an equation and solve)
What are the two forms of the scalar (or dot) product?	$\underline{a} \cdot \underline{b} = \lvert\underline{a}\rvert\lvert\underline{b}\rvert \cos\theta$ (for this version, remember that the vectors must NOT be "nose-to-tail") $\underline{a} \cdot \underline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$
How do you find the angle between two vectors?	Use the dot product and solve to find θ - or use formula $\cos\theta = \dfrac{\underline{a} \cdot \underline{b}}{\lvert\underline{a}\rvert\lvert\underline{b}\rvert}$
How do you show that two vectors are perpendicular?	Show that $\underline{a} \cdot \underline{b} = 0$
Useful rules: $\underline{a} \cdot \underline{a} =$ **?** $\underline{a} \cdot (\underline{b} + \underline{c}) =$ **?**	 $\lvert\underline{a}\rvert^2$ $\underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c}$

Further Calculus	
What do you get if you differentiate: $\sin x$ **?** $\cos x$ **?**	 $\cos x$ $-\sin x$
What do you get if you integrate: $\sin x$ **?** $\cos x$ **?**	 $-\cos x + c$ $\sin x + c$
What is the chain rule for differentiation? **(How do you differentiate** $f(g(x))$ **?)**	$f'(g(x)) \times g'(x)$ (differentiate around the brackets, then multiply by the derivative of what is inside the brackets)
What do you get if you differentiate: $\sin(ax+b)$ **?** $\cos(ax+b)$ **?**	 $a\cos(ax+b)$ $-a\sin(ax+b)$
What do you get if you integrate: $\sin(ax+b)$ **?** $\cos(ax+b)$ **?**	 $-\frac{1}{a}\cos(ax+b)+c$ $\frac{1}{a}\sin(ax+b)+c$
What you get if you integrate $(ax+b)^n$ **?**	$\frac{(ax+b)^{n+1}}{a(n+1)}+c$

The Wave Function	
How do you express $a\cos x + b\sin x$ in the form $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$?	Expand the brackets Equate coefficients Solve to find k (square and add to get k^2) Solve to find α (divide to get $\tan\alpha$)
How do you know which quadrant α is in?	Look at the signs for $k\cos\alpha$ and $k\sin\alpha$ - if both are positive then α is acute, otherwise you need to do an ASTC diagram
Given the choice, which version of the wave function should you use?	If it starts with cos, use $k\cos(x \pm \alpha)$ If it starts with sin, use $k\sin(x \pm \alpha)$ Use the version which keeps both coefficients positive, if possible
How do you find the maximum or minimum values of a wave function	Think of the graph: when is cos (or sin) at a maximum or minimum, then adjust as necessary
How do you solve $a\cos x + b\sin x = c$?	Put the left-hand side into a wave function form, then solve in the usual way
What if the question has $2x$ or $3x$ etc?	You still solve the problems in the usual way with k and α found as before – but at the end you will need to divide any answers to find x

Exponential and Logarithmic Functions	
What points does the graph of $y = a^x$ always pass through? (y equals a to the power x)	$(0,1)$ and $(1,a)$
What points does the graph of $y = \log_a x$ always pass through? (y equals the log of x, base a)	$(1,0)$ and $(a,1)$
How do you rewrite $y = \log_a x$ in power form?	$x = a^y$
How do you solve an equation where x is the power? (eg $4^x = 10$)	Take logs of both sides then use log rules to work out x
How do you solve a log equation?	Express each side as a single log then "cancel" the logs Or, get logs to one side and numbers to the other, then rewrite using power form
Log rules: $\log x + \log y =$ **?** $\log x - \log y =$ **?** $\log x^n =$ **?** $\log_a 1 =$ **?** $\log_a a =$ **?**	 $\log xy$ $\log \frac{x}{y}$ $n \log x$ 0 1
How do you get log to base e (the natural log) on your calculator?	ln button
How do you get log to base 10 on your calculator?	log button
If the graph of $\log y$ against $\log x$ is a straight line, how do you find y in terms of x?	$y = kx^n$ The values of k and n can be found from the graph
If the graph of $\log y$ against x is a straight line, how do you find y in terms of x?	$y = ab^x$ The values of a and b can be found from the graph

Revision from Credit/Intermediate 2	
What is the Sine Rule?	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$
What is the Cosine Rule?	$a^2 = b^2 + c^2 - 2bc\cos A$
What is the formula for the area of a triangle?	$Area = \frac{1}{2}ab\sin C$
How do you solve a quadratic equation?	Make one side zero, then factorise the other (or use the quadratic formula)
How do you factorise a quadratic?	Look for: 1. Common factor 2. Difference of squares 3. Double brackets
What is the quadratic formula?	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
How do you find the solution to a simple trig equation, with solutions from 0 to 360 degrees?	Find the acute angle then use an ASTC diagram to find the solutions (usually two)
How do you write $\sqrt{x}$ as a power of x?	$x^{\frac{1}{2}}$ (x to the power a half)
How do you write $\sqrt[n]{x^m}$ as a power of x? (the nth root of x to the power m)	$x^{\frac{m}{n}}$
What is x^0?	1
How do you write x^{-n} with a positive power?	$\frac{1}{x^n}$