Logs and Exponentials (Powers) KeyPoints

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Powers and logs are related by the following rule.

$$a^x = y \qquad\qquad \log_a(y) = x$$

The easiest way of thinking about it is "what power of a gives y ".

1. From the above we can derive the following rules.

$$a^m \cdot a^n = a^{(m+n)} \qquad \log_a(xy) = \log_a(x) + \log_a(y)$$

$$\frac{a^m}{a^n} = a^{m-n} \qquad \log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$$

$$\left(a^m\right)^n = a^{m \cdot n} \qquad \log_a\left(x^n\right) = n \log_a(x)$$

Use your calculator to convince yourself that these rules do hold.

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2. Some of the values you should know instantly are:-

$a^0 = 1$ $\log_a(1) = 0$ log of 1 to any base is 0

$a^1 = a$ $\log_a(a) = 1$ log of a to base a is 1

There is a special number e=2.718....., which is given a special name called the natural exponential function y=exp(x) and the corresponding log function is called the natural log.

A point worth noting is that the differential of y=exp(x) is y'=exp(x) i.e. the same function!

This function will come up time and time again if you go on to study maths to an advanced level.

3. We can use the log and power rules to find values of functions of the form.

$y = a \cdot x^b$ and $y = a \cdot b^x$

We transform these functions using the log/power rules into straight line functions, find the values for a,b and then transform them back again using the log/power rules.

$y = a \cdot x^b$ $\log(y) = \log(a) + b \cdot \log(x)$

$y = a \cdot b^x$ $\log(y) = \log(b) \cdot x + \log(a)$

It should become clear after an example of both.