

Logs and Exponential (Powers) Examples

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

Q1. Simplify the following:-

$$a^4 \cdot a^7$$

$$\log_{10}(2) + \log_{10}(32)$$

$$\frac{a^7}{a^4}$$

$$\log_{10}(32) - \log_{10}(2)$$

$$(a^4)^7 \cdot a^5$$

$$\log_{10}(2) + \log_{10}(40) - \log_{10}(8)$$

Solution

$$a^4 \cdot a^7 = a^{(4+7)} = a^{11}$$

$$\log_{10}(2) + \log_{10}(32) = \log_{10}(64)$$

$$\frac{a^7}{a^4} = a^{7-4} = a^3$$

$$\log_{10}(32) - \log_{10}(2) = \log_{10}\left(\frac{32}{2}\right) = \log_{10}(16)$$

$$(a^4)^7 \cdot a^5 = a^{7 \cdot 4} \cdot a^5 = a^{28+5} = a^{33}$$

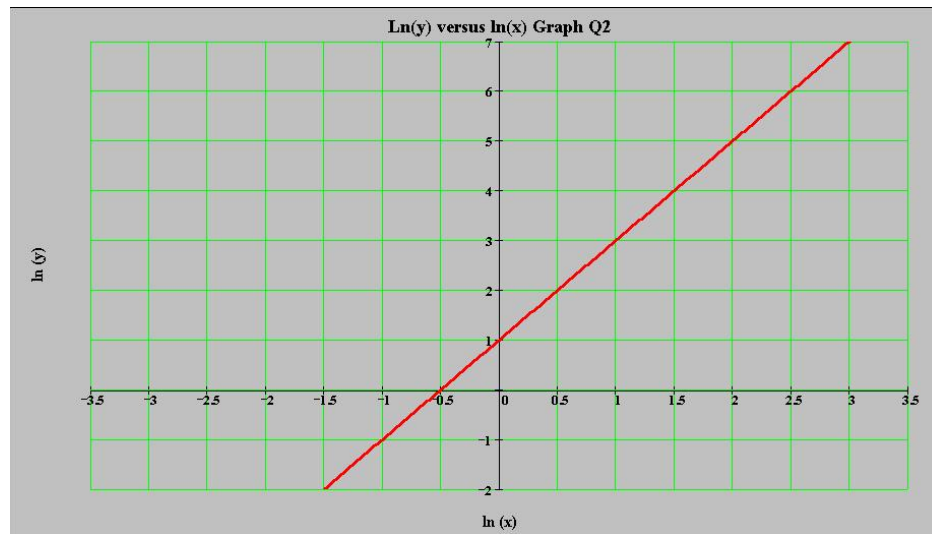
$$\log_{10}(2) + \log_{10}(40) - \log_{10}(8) = \log_{10}\left(\left(\frac{2 \cdot 40}{8}\right)\right) = \log_{10}(10) = 1$$

Logs and Exponential (Powers) Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q2. We suspect the 2 parameters x, y are related by the following relationship.

$$y = a \cdot x^b$$



Using the graph provided determine if this relationship seems reasonable (explain why!) and find values for a and b and write out the full equation.

Logs and Exponential (Powers) Examples

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

Solution

Since the graph is a straight line of the form $\log_e(y) = b \cdot \log_e(x) + \log_e(a)$ then it is reasonable to assume the relationship above.

From the graph the gradient is $b=2$ and the constant is $\log_e(a) = 1$.

Hence the equation is

We have the form

$$\log_e(y) = \log_e(a) + b \cdot \log_e(x)$$

$$b = 2 \quad \text{and} \quad \log_e(a) = 1 \quad \text{therefore} \quad a = e$$

We have

$$\log_e(y) = \log_e(e) + 2 \cdot \log_e(x)$$

taking antilogs of both sides and using log/power rules we have

$$y = e \cdot x^2$$

Logs and Exponential (Powers) Examples

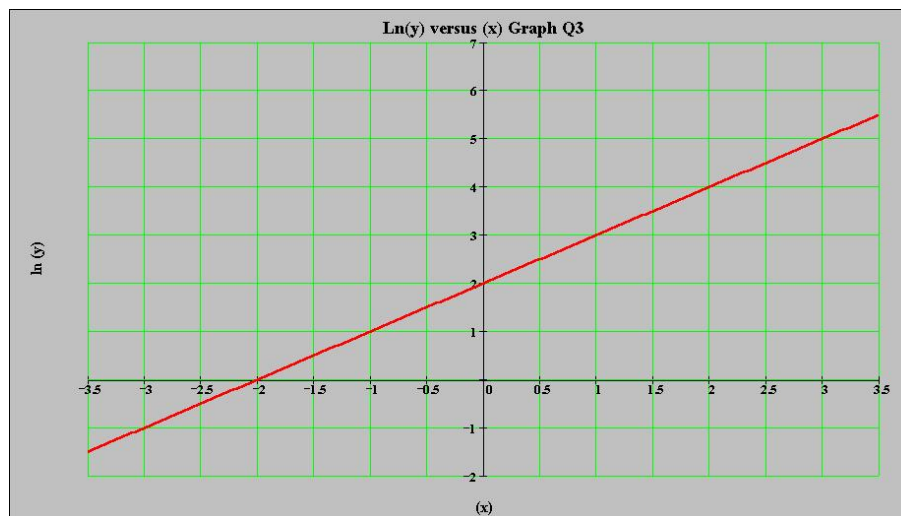
Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

Q3. We suspect the 2 parameters x, y are related by the following relationship.

$$y = a \cdot b^x$$

Using the graph provided determine if this relationship seems reasonable (explain why!) and find values for a and b and write out the full equation.



Logs and Exponential (Powers) Examples

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

Solution

Since the graph is a straight line of the form $\log_e(y) = \log_e(b) \cdot (x) + \log_e(a)$ then it is reasonable to assume the relationship above.

From the graph the gradient is $\log_e(b)=1$ and the constant is $\log_e(a) = 2$.

Hence the equation is

We have the form

$$\log_e(y) = \log_e(a) + \log_e(b) \cdot (x)$$

$$\log_e(b) = 1 \quad \text{and} \quad \log_e(a) = 2 \quad \text{therefore} \quad b = e \quad a = e^2$$

We have

$$\log_e(y) = \log_e(e^2) + \log_e(e) \cdot x$$

taking antilogs of both sides and using log/power rules we have

$$y = e^2 \cdot e^x = e^{2+x}$$