Quadratic Theory Examples

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1. State which of the following functions are polynomial functions justifying your answer, stating the degree of the function where appropriate.

$x^3 + x^2 + 2x + 1$

$x^{-2} + 1$

$x^3 + x^4 + \frac{1}{2}x^6 + 5$

Solution

$x^3 + x^2 + 2x + 1$ Polynomial of degree 3

$x^{-2} + 1$ Not polynomial since -2 is a negative whole number

$x^3 + x^4 + \frac{1}{2}x^6 + 5$ Polynomial of degree 6

2. Find the roots of the following quadratic functions below.

$x^2 - 5x + 6$:

$x^2 - 20x + 100$

Solution

$x^2 - 5x + 6 = (x-3)(x-2) = 0 \quad x = 3 \quad x = 2$

$x^2 - 20x + 100 = (x-10)(x-10) = 0 \quad x = 10$

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3. By finding the discriminant of the quadratic equations below determine the number roots each function has. (you can evaluate them if you wish if you need the practice !)

$x^2 - 1 = 0$ $x^2 + 1 = 0$ $x^2 + 2x + 1 = 0$

Solution

$x^2 - 1 = 0$	$b^2 - 4ac = 0^2 - [4 \cdot 1 \cdot (-1)] = 4 > 0$	Hence 2 real roots
$x^2 + 1 = 0$	$b^2 - 4ac = 0^2 - (4 \cdot 1 \cdot 1) = -4 < 0$	Hence no real roots
$x^2 + 2x + 1 = 0$	$b^2 - 4ac = (-2)^2 - (4 \cdot 1 \cdot 1) = 0$	Hence one real root

4. Is the line with equation given below a tangent to the circle with equation given below?

$y = x + 8$ $x^2 + y^2 - 4x - 16y + 66 = 0$

Solution

We solve

$(x + 8) = x^2 + y^2 - 4x - 16y + 66 = 0$

$x^2 + (x + 8)^2 - 4x - 16(x + 8) + 66 = 0$

$2x^2 - 4x + 2 = 0$

The discriminant is

$b^2 - 4 \cdot ac = 16 - 4 \cdot 2 \cdot 2 = 0$ one real root

Since there is only one real root the line is a tangent to the circle.

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5. Solve the quadratic inequality given below.

$x^2 - 5x + 6 > 0$

Solution

$x^2 - 5x + 6 = (x - 3)(x - 2) = 0 \qquad x = 3 \qquad x = 2$

$x > 3 \qquad x^2 - 5x + 6 > 0 \qquad 4^2 - 5 \cdot 4 + 6 = 2$

$x < 2 \qquad x^2 - 5x + 6 > 0 \qquad 0^2 - 5 \cdot 0 + 6 = 6$

Between

$x = 3 \qquad x = 2 \qquad x^2 - 5x + 6 < 0$

eg x = 2.5

$(2.5)^2 - 5 \cdot (2.5) + 6 = -0.25$

Hence solution is $x < 2$ and $x > 3$

6. Find a quadratic equation that satisfies the given roots.

$x = 5 \qquad \text{and} \qquad x = 8$

Solution

Hence we have

$(x - 8)(x - 5) = 0$

$x^2 - 13x + 40 = 0$

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7. Given that the function below is continuous show that it has a root between -4 and -2.

$f(x) = x^3 + 27$

Solution

$f(x) = x^3 + 27$

$f(-4) = (-4)^3 + 27 = -37$ Hence $f(x) < 0$

$f(-2) = (-2)^3 + 27 = 19$ Hence $f(x) > 0$

Hence there is a root between -4 and -2 since f(x) changes between negative to positive.