Vector Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q1. Which of these quantities are vectors?

Speed, distance, time, velocity, force

Solution

Speed - not a vector has magnitude only.

Distance - not a vector has magnitude only.

Time - not a vector has magnitude only.

Velocity - is a vector it has magnitude and direction.

Force - is a vector it has magnitude and direction.

Q2. Find the magnitude of the vector below.

$$\underline{v} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$$

Solution

$$|\underline{v}| = \sqrt{(3)^2 + (0)^2 + (4)^2} = 5$$

Vector Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q3. Are the following vectors equal?

$$\underline{v} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \qquad \underline{w} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Solution

No they have the same magnitude of 1, but their directions are different, v points along the x-axis and w points along the y-axis.

Q4. Find a+b and a-b for the vectors below.

$$\underline{a} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

Solution

$$\underline{a} + \underline{b} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ 4 \\ 2 \end{pmatrix} \qquad \underline{a} + \underline{b} = \begin{pmatrix} 6 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$$

Vector Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q5. Find the negative of the vector below.

$$\underline{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

Solution

$$\underline{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \qquad -\underline{v} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$$

Q6. Find the scalar multiple of the vector below when k= 1/2 and k= -2. Describe the effect that k has on the vector in each case.

Solution

$$\underline{v} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \underline{v} = \frac{1}{2}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ 0 \end{pmatrix} \qquad -2\underline{v} = -2\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix}$$

Vector Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q7. Are the points A (1, 1), B (2, 2) and C (4, 4) collinear, explain your answer.

Solution

If collinear then AB = kAC

$$\overrightarrow{AB} = k \cdot \overrightarrow{AC}$$

$$\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} = k \cdot \left[\begin{pmatrix} 4 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right]$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = k \cdot \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 3 \\ 3 \end{pmatrix}$$

Vector Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q8. Find the mid-point m between the vectors below.

$$\underline{a} = \begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 2 \\ 10 \\ 12 \end{pmatrix}$$

Solution

$$m = 1/2\ (\underline{a}+\underline{b})$$

$$\underline{m} = \frac{1}{2} \cdot \left[\begin{pmatrix} 2 \\ 4 \\ 8 \end{pmatrix} + \begin{pmatrix} 2 \\ 10 \\ 12 \end{pmatrix} \right] = \begin{pmatrix} 2 \\ 7 \\ 10 \end{pmatrix}$$

Vector Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q9. Find the coordinates of the point P if it splits A (3, 3, 6) and B (6, 12, 18) in the ratio 1:2.

Solution

$$2\overrightarrow{AP} = \overrightarrow{PB}$$

$$2(\underline{p} - \underline{a}) = (\underline{b} - \underline{p})$$

$$3\underline{p} = \underline{b} - 2\underline{a}$$

$$\underline{p} = \frac{\underline{b} - 2\underline{a}}{3}$$

$$\underline{p} = \frac{\begin{pmatrix} 6 \\ 12 \\ 18 \end{pmatrix} - 2\begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}}{3}$$

$$\underline{p} = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$$

Vector Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q10. Find the angle between the vectors below.

$$\underline{a} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$$

Solution

$$|a| = \sqrt{(1)^2 + (0)^2 + (2)^2} = \sqrt{5}$$

$$|b| = \sqrt{(2)^2 + (3)^2 + (5)^2} = \sqrt{38}$$

$$a \cdot b = [(1 \cdot 2) + (0 \cdot 3) + (2 \cdot 5)] = 12$$

$$\cos\theta = \frac{12}{\sqrt{5} \cdot \sqrt{38}} = 0.871$$

$$\theta = 29.4 \text{degrees}$$

Vector Examples

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

Q11. Are the vectors below perpendicular to each other?

$$\underline{a} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$$

Solution

$$\underline{a} \cdot \underline{b} = [(0 \cdot 0) + (1 \cdot 0) + (0 \cdot 4)] = 0$$

Vectors are perpendicular since the scalar/dot product $\underline{a}.\underline{b} = \underline{0}$

Q12. Express the vector below in terms of the base vectors $\underline{i}$, $\underline{j}$, and $\underline{k}$.

$$\underline{w} = \begin{pmatrix} 13 \\ -5 \\ -2 \end{pmatrix}$$

Solution

$$\underline{w} = \begin{pmatrix} 13 \\ -5 \\ -2 \end{pmatrix}$$

$$\underline{w} = 13\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} - 5\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} - 2\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 13 \\ -5 \\ -2 \end{pmatrix}$$

$$\underline{w} = 13 \cdot \underline{i} - 5 \cdot \underline{j} - 2 \cdot \underline{k}$$