

Wave Function KeyPoints

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

Given any function of the form below

$$y = a \cdot \sin(x) + b \cdot \cos(x) \quad \text{or} \quad y = a \cdot \cos(x) + b \cdot \sin(x)$$

We can rearrange it into one of the forms below which is called the Wave function.

$$y = A \sin(x \pm \alpha) \quad \text{or} \quad y = A \cos(x \pm \alpha)$$

A = The **A**mplitude (maximum height from the reference axis) of the sine wave.

α = shifts the standard sine α graph units to the right if negative and to the left negative.

Why do this?

It makes it far easier to sketch / find the key information about the function.

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To convert to the standard wave form follow the procedure below:-

Step 1

Choose the required form you want to express it in:-

$$y = A\sin(x \pm \alpha) \text{ or } y = A\cos(x \pm \alpha)$$

Step 2

Expand the appropriate form in step 1 above to one of the following:-

$$A\sin(x \pm \alpha) = A\sin(x) \cdot \cos(\alpha) \pm A\cos(x) \cdot \sin(\alpha)$$

$$A\cos(x \pm \alpha) = A\cos(x) \cdot \cos(\alpha) \mp A\sin(x) \cdot \sin(\alpha)$$

Step 3

Compare the co-efficients of $\cos(x)$ and $\sin(x)$ in step 2 with the original equation.

Depending on the form used

$$A\cos(\alpha) = a \quad \text{and} \quad A\sin(\alpha) = b$$

or

$$A\cos(\alpha) = b \quad \text{and} \quad A\sin(\alpha) = a$$

Note a and b can be positive or negative.

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Step 4

Divide equations found in step 3 to get the value for α

Depending on the form used

$$\frac{\sin(\alpha)}{\cos(\alpha)} = \tan(\alpha) = \frac{b}{a} \quad \text{Hence} \quad \alpha = \tan^{-1}\left(\frac{b}{a}\right)$$

or

$$\frac{\sin(\alpha)}{\cos(\alpha)} = \tan(\alpha) = \frac{a}{b} \quad \text{Hence} \quad \alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

Remember depending on the signal of a and b determines which quadrant α lies!!!)

Step 5

Square the equations in step 3 and then add them and finally take the square root gives (A).

(Sounds hard but quite easy really!!)

$$A^2 \cdot \sin^2 x + A^2 \cdot \cos^2 x = a^2 + b^2$$

$$A^2 \cdot (\sin^2 x + \cos^2 x) = a^2 + b^2$$

$$A = \sqrt{(a^2 + b^2)}$$

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Step 6

Write down the wave equation in standard form you selected initially.

$$y = A\sin(x \pm \alpha) \quad \text{or} \quad y = A\cos(x \pm \alpha)$$