Trig. Advance Examples

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Graduate Bsc (Hons) MathsSci (Open) GIMA

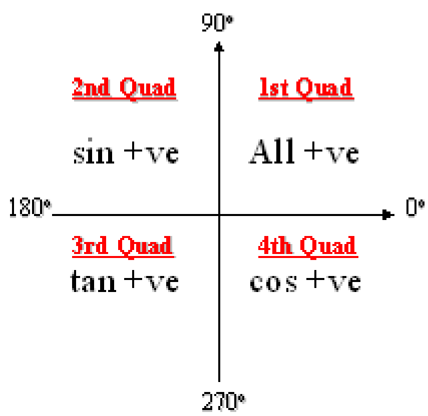
1. Find the values for x in the range $0^o \le x \le 360^o$

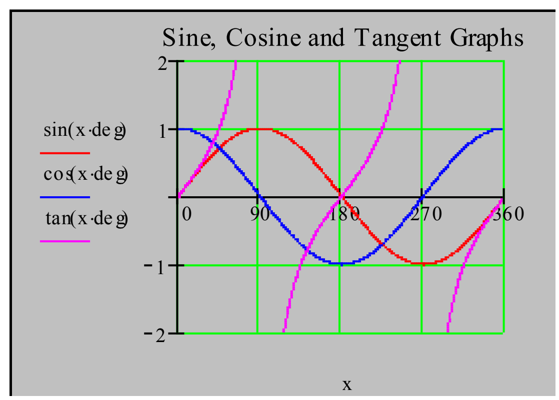
(a) $1 + \sqrt{2}\cdot\cos x^o = 0$

(b) $\tan^2 x - 3 = 0$

Solution

Remembering!!!





(a) $1 + \sqrt{2}\cdot\cos x = 0$ $\quad \sqrt{2}\cdot\cos x = -1$ $\quad \cos x = \frac{-1}{\sqrt{2}}$ $\quad x^o = \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)$

$x^o = (180 - 45) = 135^o$ 2nd quad $\qquad x^o = (180 + 45) = 225^o$ 3rd quad

(b) $\tan^2 x - 3 = 0$

$\tan^2 x = 3$ $\qquad \tan x = \sqrt{3}$ $\qquad x^o = \tan^{-1}(\sqrt{3})$

$x^o = 60^o$ 1st quad $\qquad x^o = (180 + 60) = 240^o$ 3 rd quad

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2. Solve for x in the range $0 \le x \le 360^o$

(a) $\sin 2x = -1$

(b) $\tan^2 2x = 1$

Solution

(a) $\sin 2x = -1$ $\quad 2x = \sin^{-1}(-1)$ $\quad 2x = (270) + (270 + 360) + \ldots$

$x = 135^o$ $\quad x = 315^o$

(b) $\tan^2 2x = 1$ $\quad 2x = \tan^{-1}(1)$ $\quad 2x = (45) + (45 + 360) + \ldots$ 1st Quad

and

$2x = (225) + (225 + 360) + \ldots$ 3rd Quad

$x = 22.5^o$ $\quad x = 202.5^o$ $\quad x = 112.5^o$ $\quad x = 236.5^o$

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3. Find the coordinates of intersection of the line and function given the equations below.

$y = 0$ $y = \sin x$ $0^o \le x \le 360^o$

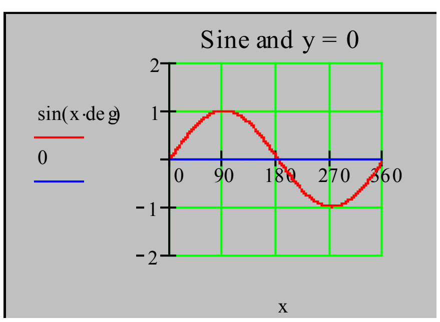
Solution

At the point of intersection we have

$\sin x = 0$ $x = \sin^{-1}(0)$ $x = 0^o$ $x = 180^o$ $x = 360^o$

Hence coordinates are

$(0^o, 0)$ $(180^o, 0)$ $(360^o, 0)$



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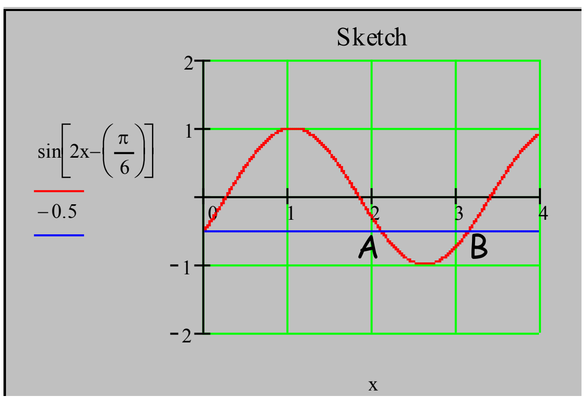
4. Find the coordinates of intersection A and B from the diagram below.

Note scale is in radians.

$0 \le x \le 4 \cdot \text{radians}$



<u>Solution</u>

The y coordinate in both cases is simply

$y = -0.5$

The x coordinate is found by

$\sin\left(2\cdot x - \frac{\pi}{6}\right) = -0.5$ $\quad 2\cdot x - \frac{\pi}{6} = \sin^{-1}(-0.5)$ $\quad 2\cdot x - \frac{\pi}{6} = \left(\frac{7\cdot\pi}{6}\right)$ 3rd Quad $\quad 2\cdot x - \frac{\pi}{6} = \left(\frac{11\cdot\pi}{6}\right)$ 4th Quad

$2\cdot x = \frac{7\cdot\pi}{6} + \frac{\pi}{6} = \frac{8\cdot\pi}{6} = \frac{4\pi}{3}$ $\quad 2\cdot x = \frac{11\cdot\pi}{6} + \frac{\pi}{6} = \frac{12\pi}{6} = 2\cdot\pi$ $\quad x = \frac{4\cdot\pi}{6} = \frac{2\cdot\pi}{3}$

$2\cdot x = \frac{2\cdot\pi}{2} = \pi$ Hence coordinates are $A\left(\frac{2\cdot\pi}{3}, -0.5\right)$ and $B(\pi, -0.5)$

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5. Solve the following quadratics.

(a) $2\cdot\sin^2 x + 3\cdot\sin\cdot x + 1 = 0$ $\quad 0 \le x \le 360^o$

(b) $3\cdot\cos^2 x - 5\cdot\cos\cdot x - 2 = 0$ $\quad 0 \le x \le 360^o$

Solution

(a) $2\cdot\sin^2 x + 3\cdot\sin\cdot x + 1 = 0$

Let $p = \sin\cdot x$

We then have $2\cdot p^2 + 3\cdot p + 1 = 0$

Factorise $(2\cdot p + 1)\cdot(p + 1) = 0$

$p = \frac{-1}{2}$ and $p = -1$

So we have $p = \sin\cdot x = \frac{-1}{2}$ and $p = \sin\cdot x = -1$

$x = \sin^{-1}\left(\frac{-1}{2}\right)$ $\quad x = \sin^{-1}(-1)$

$x = \sin^{-1}\left(\frac{-1}{2}\right)$ $\quad x = 270^o$

$x = 150^o$ $\quad x = 210^o$

Hence solutions are

$x = 150^o, 210^o, 270^o$

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5. (b) $3 \cdot \cos^2 x - 5 \cdot \cos \cdot x - 2 = 0$ $\quad 0 \le x \le 360^o$

Solution

Let $p = \cos \cdot x$

We then have $3 \cdot p^2 - 5 \cdot p - 2 = 0$

Factorise $(3p + 1) \cdot (p - 2) = 0$

$p = \frac{-1}{3}$ and $p = 2$

So we have

$p = \cos \cdot x = \frac{-1}{3}$ and $p = \cos \cdot x = 2$

$x = \cos^{-1}\left(\frac{-1}{3}\right)$ $\quad$ $x = \text{no solution}$

$x = \cos^{-1}\left(\frac{-1}{3}\right)$ $\quad$ Cosine can only take values of -1 to +1. !!

$x = 109.5^o$ and $x = 250.5^o$

Hence solutions are

$x = 109.5^o, 250.5^o$

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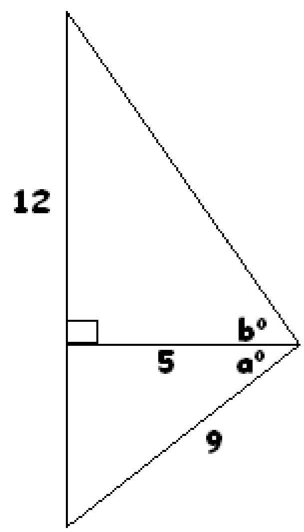
6. From the diagram below prove that:-

$$\sin(a + b) = \frac{10\cdot\sqrt{14} + 60}{117}$$



Solution

$$\sin(a + b) = \sin\cdot a\cdot\cos\cdot b + \cos\cdot a\cdot\sin\cdot b$$

$$\frac{\left(\sqrt{9^2 - 5^2}\right)}{9}\cdot\frac{5}{\sqrt{\left(12^2 + 5^2\right)}} + \frac{5}{9}\cdot\frac{12}{\sqrt{\left(12^2 + 5^2\right)}}$$

$$\frac{\sqrt{56}}{9}\cdot\frac{5}{13} + \frac{5}{9}\cdot\frac{12}{13}$$

$$\frac{5\cdot\sqrt{4}\cdot\sqrt{14} + 60}{117}$$

$$\frac{10\cdot\sqrt{14} + 60}{117}$$

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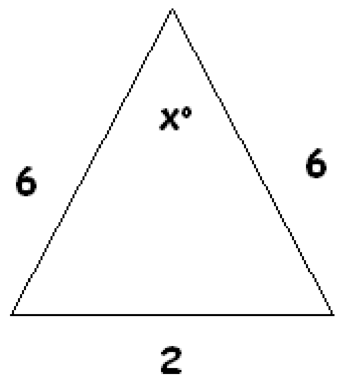
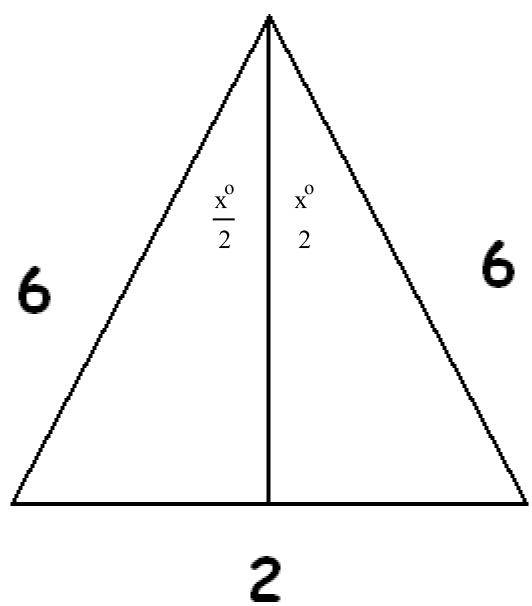
Q7 From the diagram find the exact value of sinx:-



Solution

Using double angle formula we have

$$\sin \cdot x = 2 \cdot \sin \cdot \left(\frac{x}{2}\right) \cdot \cos \cdot \left(\frac{x}{2}\right)$$



$$\sin \cdot x = 2 \cdot \frac{1}{6} \cdot \frac{\sqrt{6^2 - 1^2}}{6} = \frac{1}{3} \cdot \frac{\sqrt{35}}{6} = \frac{\sqrt{35}}{18}$$