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1. To find the equation that passes through (-1,4)
And is parallel to 3x – y + 2 = 0 we have:

Since line is parallel to $3x - y + 2 = 0$

$$y = 3x + 2$$

$$m = 3$$

Hence equation is

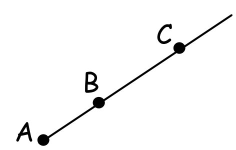
$$y - 4 = m(x - (-1)) \qquad (-1,4)$$

$$y - 4 = 3(x + 1)$$

$$y = 3x + 7$$

2. Given the diagram and that A and B are (-2, 1, -1) and (1, 3, 2).
Given ABC are collinear and related by BC = 2AB. Then C is given by:

$$AB = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} - \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} \qquad BC = 2 \times \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix}$$



$$c = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \\ 8 \end{pmatrix} \qquad Coordinates\ are\ (7,7,8)$$

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3. Given the two functions

$$f(x) = x^2 + 1 \qquad g(x) = 1 - 2x$$

(a)(i) Finding $g(f(x))$ we have:

$$g(x^2+1) = 1 - 2(x^2+1) = 1 - 2x^2 - 2 = -2x^2 - 1$$

(b) Finding $g(g(x))$ we have:

$$g(1-2x) = 1 - 2(1-2x) = 1 - 2 + 4x = 4x - 1$$

4. The range of values for k such that $kx^2 - x - 1 = 0$ l has no real roots is given by:

$$b^2 - 4ac < 0$$
$$(-1)^2 - 4 \times k \times (-1) < 0$$
$$1 + 4k < 0$$
$$4k < -1$$
$$k < -\frac{1}{4}$$

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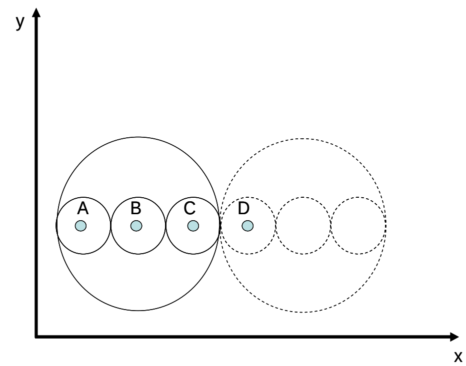
5. Given the diagram and that the small circles are congruent and their centres lie on a line parallel to the axis.

The big circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$

To find the equation of the circle D we have:



Centre $C = (-g, -f) = (7,8)$

Large Circle $r = \sqrt{((-7)^2 + (-8)^2 - 77)}$

$= \sqrt{36}$

$= 6$

Radius of small circle B $= 6 \div 3 = 2$ and centre (7,8)

Since the centres of the small circles lie on a line parallel to x-axis

Circle D has centre (15,8)

Hence equation of circle D is $(x-15)^2 + (y-8)^2 = 4$

6. Solving the equation $\sin 2x^o = 6\cos x^o \quad 0 \le x \le 360$ we get:

$\sin 2x^o = 6\cos x^o$
$2\sin x \cos x = 6\cos x$
$2\sin x \cos x - 6\cos x = 0$
$2\cos x(\sin x - 3) = 0$

$2\cos x = 0$ or $\sin x - 3 = 0$
$x = \cos^{-1}(0)$ or $\sin x = 3 \;\; (no\ solutions)$

$x = 90^0 \, and \;\; 270^0$

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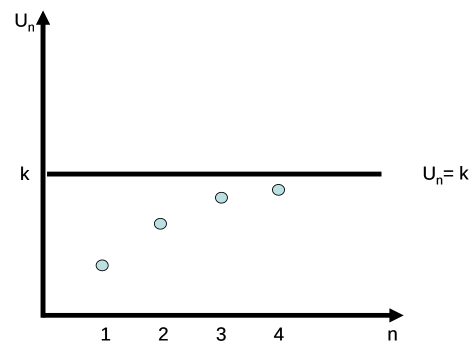
7. Given $$U_{n+1} = \frac{1}{4}U_n + 16 \qquad U_0 = 0$$

(a) Calculating the next few terms we get:



$$U_1 = \frac{1}{4} \times 0 + 16 \qquad U_1 = 16$$
$$U_2 = \frac{1}{4} \times 16 + 16 \qquad U_1 = 20$$
$$U_3 = \frac{1}{4} \times 20 + 16 \qquad U_1 = 21$$

(b)(i) This sequence has a limit because multiplier 0.25 has magnitude less that 1.

(ii) The exact find of limit k is given by:

Let k = limit value

$$k = \frac{1}{4}k + 16$$
$$k - \frac{1}{4}k = 16$$
$$k = \frac{16}{\frac{3}{4}} = 21\frac{1}{3}$$

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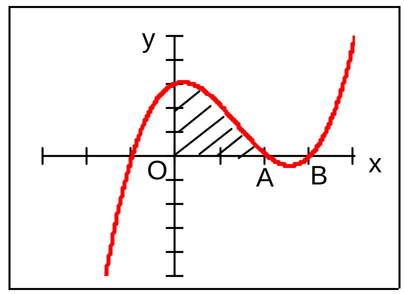
8. Given the diagram of $y = x^3 - 4x^2 + x + 6$

(a) To show graph cuts x-axis at (3, 0) we have:



$x = 3$

$y = 3^3 - 4 \times 3^2 + 3 + 6 = 0$

(b) To find coordinates of A.
Using synthetic division we get:

3	1	-4	1	6
		3	-3	-6
	1	-1	-2	0

$$x^3 - 4x^2 + x + 6 = (x-3)(x^2 - x - 2) = (x-3)(x-2)(x+1)$$

Using the graph the coordinates of A are (2, 0).

(c) The shaded area is :

$$\int_0^2 (x^3 - 4x^2 + x + 6)dx = \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x\right]_0^2$$

$$= \left[\frac{2^4}{4} - \frac{4 \times 2^3}{3} + \frac{2^2}{2} + 6 \times 2\right] - 0$$

$$= 4 - \frac{32}{3} + 2 + 12$$

$$= \frac{54}{3} - \frac{32}{3}$$

$$= \frac{22}{3} = 7\frac{1}{3}$$

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9. Given the function $f(x) = 3x - x^3$

(a) The exact values where the function crosses x and y-axis's are given by:

$x = 0$

$y = 3 \times 0 - 0^3 = 0$

$y - axis \quad (0,0)$

$y = 0$

$3x - x^3 = 0$

$x(3 - x^2) = x(\sqrt{3} - x)(\sqrt{3} + x) = 0$

$x - axis \quad (0,0)\ ,\ (\sqrt{3},0)\ ,\ (-\sqrt{3},0)$

(b) Finding the coordinates of the stationary points and their nature:

$f'(x) = 3 - 3x^2$

Stationary pt $f'(x) = 0$

$3 - x^2 = 3(1 - x^2) = 3(1 - x)(1 + x) = 0$

$x = 1$ and $x = -1$

$x = 1 \quad y = 3 \times 1 - 1^3 = 2 \qquad (1,2)$

$x = -1 \quad y = 3 \times (-1) - (-1)^3 = -2 \qquad (-1,-2)$

Nature is: Constructing a nature table for the stationary point we see that (-1, -2) is a minimum and (1, 2) is a maximum.

x	-2	-1	0	1	2
f'(x)	+	0		0	+
	\	—	/	—	\

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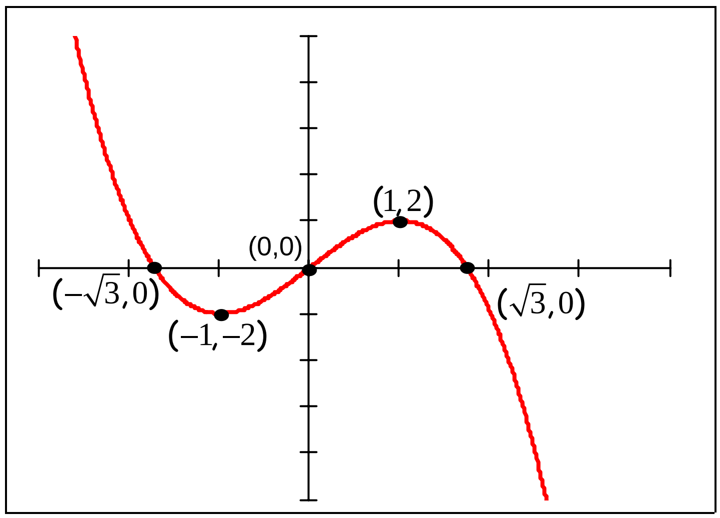
9. (c) Sketching the function $f(x) = 3x - x^3$



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11. (a) Expressing $f(x) = \sqrt{3}\cos x + \sin x$ in the form

$$k\cos(x-a) \quad k>0 \quad 0<a<\frac{\pi}{2}$$

$$k\cos x\cos a + k\sin x\sin a = \sqrt{3}\cos x + \sin x$$

$$k = \sqrt{\sqrt{3}^2 + 1^2} = 2$$

$$\tan a = \frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

$$\sqrt{3}\cos x + \sin x = 2\cos\left(x - \frac{\pi}{6}\right)$$

(b) Sketching the graph of $y = f(x)$ $0 \le x \le 2\pi$ in part A: