

Higher Still Level Paper 1 Solutions 2006

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

1. Given the diagram.

(a) The equation of the median BD is:

$$D = \left[\frac{(-1) + 7}{2}, \frac{12 + (-2)}{2} \right] = (3, 5)$$

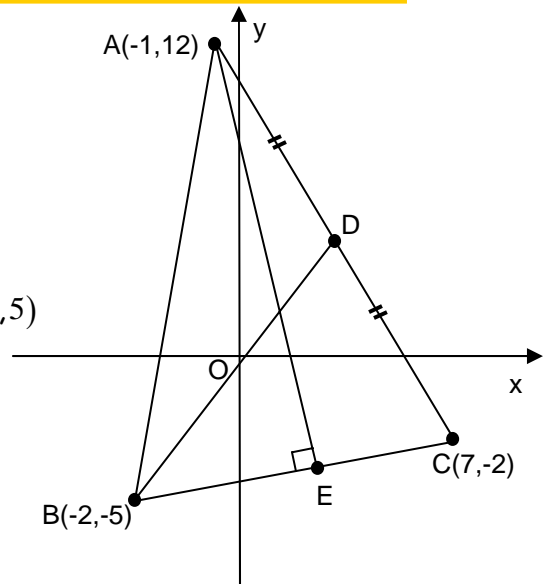
$$\text{Gradient } m_{BD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-5)}{3 - (-2)} = 2$$

Hence equation is

$$y - b = m(x - a) \quad D(3, 5)$$

$$y - 5 = 2(x - 3)$$

$$y = 2x - 1$$



(b) Equation of altitude AE is:

$$\text{Gradient } m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-5)}{7 - (-2)} = \frac{3}{9} = \frac{1}{3}$$

Using the perpendicular formula

$$m_{AE} = -3$$

Hence equation of altitude AE is

$$y - b = m(x - a) \quad A(-1, 12)$$

$$y - 12 = -3(x - (-1))$$

$$y = -3x + 9$$

Higher Still Level Paper 1 Solutions 2006

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

1. (c) Coordinates of the point of intersection of BD and AE are:

At the point of intersection we have,

$$2x - 1 = -3x + 9$$

$$5x = 10$$

$$x = \frac{10}{5} = 2 \quad ; \quad y = 2(2) - 1 = 3$$

Coordinates are (2,3)

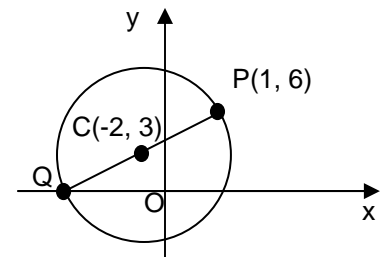
2. (a) The equation of the circle is:

$$\text{Radius } r = \sqrt{(1 - (-2))^2 + (6 - 3)^2} = \sqrt{18}$$

$$\text{Centre } C = (-2, 3)$$

$$\text{Equation is: } (x - (-2))^2 + (y - 3)^2 = \sqrt{18}$$

$$(x + 2)^2 + (y - 3)^2 = \sqrt{18}$$



- (c) Gradient of PQ is: By symmetry of the circle Q is (-5, 0)

$$\text{Gradient } m_{CQ} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 0}{-2 - (-5)} = \frac{3}{3} = 1$$

Using the perpendicular formula

$$m_{\text{tangent}} = -1$$

Hence equation of tangent at Q is

$$y - b = m(x - a) \quad Q(-5, 0)$$

$$y - 0 = -1(x - (-5))$$

$$y = -x - 5$$

Higher Still Level Paper 1 Solutions 2006

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

3. Given the two functions

$$f(x) = 2x + 3 \quad g(x) = 2x - 3$$

- (a)(i) Finding $f(g(x))$ we have:

$$f(2x - 3) = 2(2x - 3) + 3 = 4x - 3$$

- (ii) Finding $g(f(x))$ we have:

$$g(2x + 3) = 2(2x + 3) - 3 = 4x + 3$$

- (b) Least possible value of $f(g(x)) \times g(f(x))$

$$\begin{aligned} f(g(x)) \times g(f(x)) &= (4x - 3)(4x + 3) \\ &= 16x^2 - 9 \end{aligned}$$

Minimum value occurs when $x = 0$

minimum value is -9

4. Given the recurrence relation $U_{n+1} = 0.8U_n + 12 \quad U_0 = 4$

- (a) This sequence has a limit because multiplier 0.8 has magnitude less than 1.

- (b) The Limit is: Let L = limit value

$$L = 0.8L + 12$$

$$L = \frac{12}{(1 - 0.8)} = 60$$

Higher Still Level Paper 1 Solutions 2006

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

5. Given the function $f(x) = (2x - 1)^5$. The stationary point is found by:

$$f'(x) = 5 \times (2x - 1)^4 \times 2 = 10(2x - 1)^4$$




Stationary pt $f'(x) = 0$

$$10(2x - 1)^4 = 0$$

$$(2x - 1) = 0$$

$$x = \frac{1}{2} \quad \text{Pt is } \left(\frac{1}{2}, 0\right)$$

Nature is: Constructing a nature table the stationary point is a rising point of inflection.

	0	1/2	1
$f'(x)$	+	0	+
			

6. Given the graph of $y = x^3 - 6x^2 + 4x + 1$

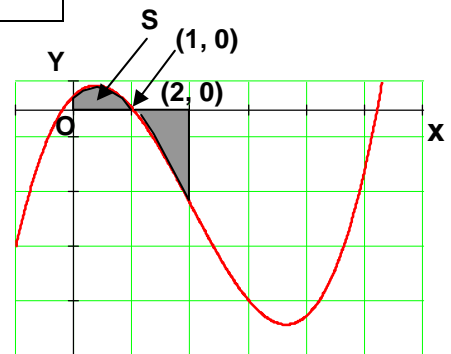
- (a) The area of the shaded area S is:

$$\int_0^1 (x^3 - 6x^2 + 4x + 1) dx = \left[\frac{x^4}{4} - 2x^3 + 2x^2 + x \right]_0^1 = 1\frac{1}{4}$$

- (b) Total area will be:

$$-\int_1^2 (x^3 - 6x^2 + 4x + 1) dx = -\left[\frac{x^4}{4} - 2x^3 + 2x^2 + x \right]_1^2 = 3\frac{1}{4}$$

$$\text{Total Area is: } 3\frac{1}{4} + 1\frac{1}{4} = 4\frac{1}{2}$$



Higher Still Level Paper 1 Solutions 2006

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

7. Solving the equation $\sin x^\circ - \sin 2x^\circ = 0$ in the interval $0 \leq x \leq 360$ we have:

$$\begin{aligned}\sin x^\circ - \sin 2x^\circ &= 0 \\ \sin x^\circ(1 - 2\cos x^\circ) &= 0\end{aligned}$$

$$\text{We have } \sin x^\circ = 0 \quad x^\circ = 0^\circ, 180^\circ, 360^\circ$$

$$\text{Also, } (1 - 2\cos x^\circ) = 0$$

$$x^\circ = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ, 300^\circ$$

$$\text{Overall solution is } x^\circ = 0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$$

8. (a) Expressing $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$ we get:

$$\begin{aligned}2x^2 + 4x - 3 &= 2(x^2 + 2x) - 3 \\ &= 2[(x + 1)^2] - 2 - 3 \\ &= 2(x + 1)^2 - 5\end{aligned}$$

- (b) The coordinates of the turning point is $(-1, -5)$

Higher Still Level Paper 1 Solutions 2006

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

9. Given $\underline{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\underline{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$

(a) Given $\underline{u} \cdot \underline{v} = 1$, then we have:

$$\begin{aligned} a_1b_1 + a_2b_2 + a_3b_3 &= 1 \\ k^3 + 3k^2 + (k+2)(-1) &= 1 \\ k^3 + 3k^2 - k - 2 &= 1 \\ k^3 + 3k^2 - k - 3 &= 0 \quad \text{as required} \end{aligned}$$

(b) To show that $(k+3)$ is a factor of $k^3 + 3k^2 - k - 3$ we use synthetic division.

-3	1	3	-1	-3
		-3	0	3
	1	0	-1	0

We have $k^3 + 3k^2 - k - 3 = (k+3)(k^2 - 1)$

Factorising we get: $(k+3)(k^2 - 1) = (k+3)(k+1)(k-1)$

(c) Since $k > 0$ then the only possible value for k is 1.

Higher Still Level Paper 1 Solutions 2006

Created by

Graduate Bsc (Hons) MathsSci (Open) GIMA

9.(d) The angle between \underline{u} and \underline{v} is:

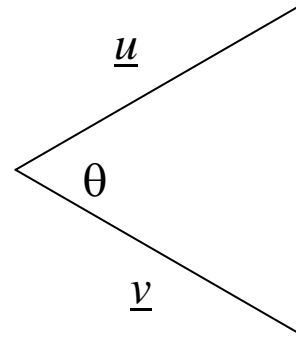
$$\underline{u} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \text{ and } \underline{v} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$$

$$|\underline{u}| = \sqrt{1^2 + 1^2 + 3^2} = \sqrt{11}$$

$$|\underline{v}| = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{11}$$

$$\underline{u} \cdot \underline{v} = 1$$

$$\cos(\theta) = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}| \cdot |\underline{v}|} = \frac{1}{\sqrt{11} \cdot \sqrt{11}} = \frac{1}{11} \text{ as required.}$$



10. Given the graph of $\log_4 y$ against x and that it obeys the law $y = a^x$. Then the value of a is:

$$y = a^x$$

$$\log_4 y = \log_4 a^x = x \log_4 a$$

$$3 = 6 \log_4 a$$

$$\frac{1}{2} = \log_4 a$$

$$4^{\frac{1}{2}} = a$$

$$a = 2$$

