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1. Given

**Put in a form that we can integrate**

$$\int \frac{4x^3 - 1}{x^2}\,dx = \int \frac{4x^3}{x^2} - \frac{1}{x^2}\,dx = \int 4x - x^{-2}\,dx$$

**Now we can integrate using the rule** $\int x^n\,dx \to \frac{x^{(n+1)}}{(n+1)}$

**We get**

$$\int 4x - x^{-2}\,dx = \frac{4x^2}{2} - \frac{x^{-1}}{-1} + C = 2x^2 + \frac{1}{x} + C$$

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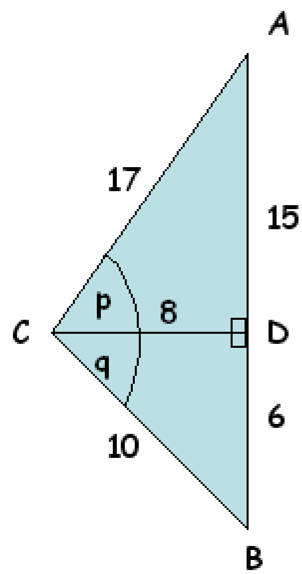
2. Given diagram




(a) $\sin(p+q) = \sin(p)\cos(q) + \cos(p)\sin(q)$

$$\frac{15}{17}\cdot\frac{8}{10} + \frac{8}{17}\cdot\frac{6}{10}$$

$$\frac{120}{170} + \frac{48}{170}$$

$$\frac{168}{170} = \frac{84}{85}$$

(b)(i) $\cos(p+q) = \cos(p)\cos(q) - \sin(p)\sin(q)$

$$\frac{8}{17}\cdot\frac{8}{10} - \frac{15}{17}\cdot\frac{6}{10}$$

$$\frac{64}{170} - \frac{90}{170}$$

$$\frac{-26}{170} = \frac{-13}{85}$$

(b)(ii) $\tan(p+q) = \dfrac{\sin(p+q)}{\cos(p+q)} = \dfrac{\frac{84}{85}}{\frac{-13}{85}} = \dfrac{-84}{13}$

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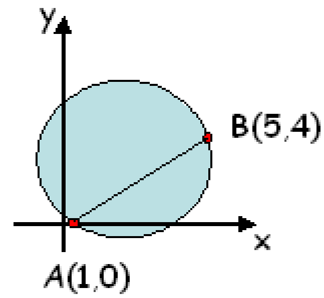
3. Given diagram




(a) The perpendicular bisector passes through the midpoint of AB.

$$\text{Midpt} = \left(\frac{1+5}{2}, \frac{4+0}{2}\right) = (3, 2)$$

The gradient of AB is given by

$$m = \frac{4-0}{5-1} = \frac{4}{4} = 1$$

The perpendicular gradient is given by

$$m_{ab} \cdot m_{pg} = -1 \qquad 1 \cdot m_{pg} = -1 \qquad m_{pg} = -1$$

So equation of perpendicular gradient is

$$y - b = m_{pg}(x - a)$$

$$y - 2 = -1(x - 3)$$

$$y - 2 = -x + 3$$

$$x + y = 5$$

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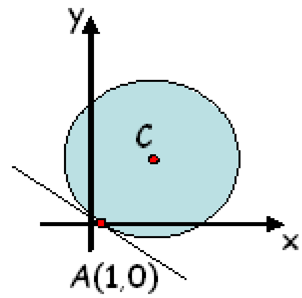


(b) Give the tangent equation

$x + 3y = 1$

Gradient is

$y = \frac{-1}{3}x + \frac{1}{3}$  $m_T = \frac{-1}{3}$

Gradient of radius line CA is

$m_T \cdot m_{CA} = -1$  $\frac{-1}{3} \cdot m_{CA} = -1$  $m_{CA} = 3$

Equation of radius line CA is

$y - b = m(x - a)$

$y - 0 = 3(x - 1)$

$3x - y = 3$

(c)(i) By solving simultaneous equations we can find co-ordinates of centre C.

$x + y = 5$  $4x = 8$  $x = 2$  $y = 5 - 2 = 3$  $C(2,3)$

$3x - y = 3$

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(ii) Equation of circle is given by

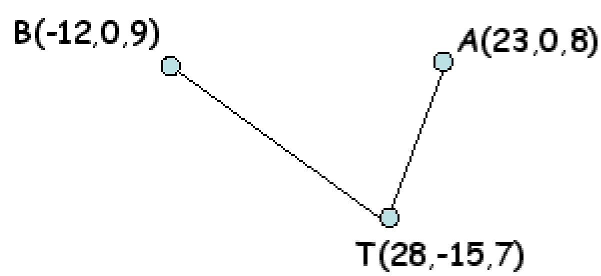
$$(x-a)^2+(y-b)^2=r^2$$

$$(a,b)=(2,3) \qquad r=\sqrt{(3-0)^2+(2-1)^2}=\sqrt{10}$$

$$(x-2)^2+(y-3)^2=10$$

4. Given diagram

(a) In component form




$$\overrightarrow{TA}=\begin{pmatrix}23\\0\\8\end{pmatrix}-\begin{pmatrix}28\\-15\\7\end{pmatrix}=\begin{pmatrix}-5\\15\\1\end{pmatrix} \qquad \overrightarrow{TB}=\begin{pmatrix}-12\\0\\9\end{pmatrix}-\begin{pmatrix}28\\-15\\7\end{pmatrix}=\begin{pmatrix}-40\\15\\2\end{pmatrix}$$

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(b) Angle between both beams is

$$\cos(\theta) = \frac{a \cdot b}{|a| \cdot |b|}$$

$$a \cdot b = [(-5) \cdot (-40) + 15 \cdot 15 + 1 \cdot 2] = 427$$

$$|a| = \sqrt{(-5)^2 + (15)^2 + 1^2} = \sqrt{251}$$

$$|b| = \sqrt{(-40)^2 + (15)^2 + 2^2} = \sqrt{1829}$$

$$\cos(\theta) = \frac{427}{\sqrt{251} \cdot \sqrt{1829}}$$

$$\cos(\theta) = 0.63$$

$$\frac{427}{\sqrt{251} \cdot \sqrt{1829}} = 0.63$$

$$\theta = 50.9^{o}$$

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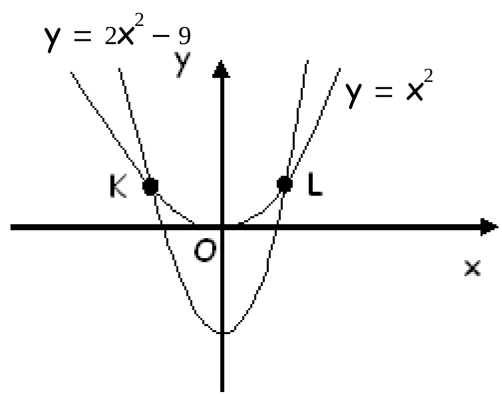
5. Given diagram




Points of intersection are given by

$2x^2 - 9 = x^2$

$2x^2 - x^2 = 9$

$x^2 = 9$

x = 3 and -3

Area between curves is given by

$$\int_{-3}^{3} x^2 - (2x^2 - 9)\,dx = \int_{-3}^{3} -(x^2) + 9\,dx = \left[-\left(\frac{x^3}{3}\right) + 9x\right]$$

$$\left[-\left(\frac{3^3}{3}\right) + 9 \times 3\right] - \left[\frac{-(-3)^3}{3} + 9\cdot(-3)\right]$$

$$[-(9) + 27] - (9 - 27)$$

$$54 - 18 = 36$$

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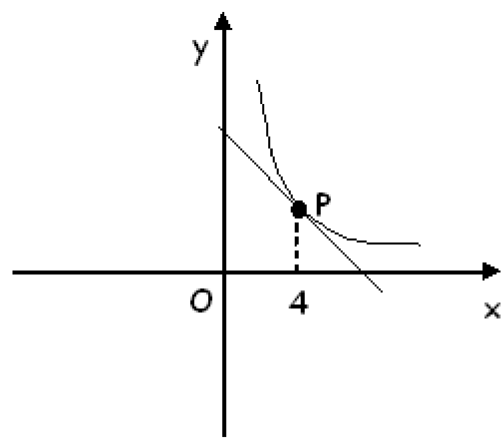
6. Given diagram




The equation of the tangent at P is found by

$$y = \frac{24}{\sqrt{x}} \qquad x > 0$$

$$y = 24x^{\frac{-1}{2}}$$

$$\frac{d}{dx}y = -12x^{\frac{-3}{2}}$$

When x = 4 the gradient is

$$\frac{d}{dx}y = -12(4)^{\frac{-3}{2}} = \frac{-12}{8} = \frac{-3}{2}$$

When x = 4 the y co-ordinate is $y = \frac{24}{\sqrt{4}} = 12$ $\qquad P(4, 12)$

Equation of tangent is

$y - b = m(x - a)$ $\qquad y - 12 = \frac{-3}{2}(x - 4)$ $\qquad 2y - 24 = -3x + 12$

$$3x + 2y = 36$$

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7. Solving using the rules for logs we get

$$\log_4(5 - x) - \log_4(3 - x) = 2 \qquad x < 3$$

$$\log_4\left(\frac{5 - x}{3 - x}\right) = 2$$

**take antilog**

$$\frac{5 - x}{3 - x} = 16$$

$$5 - x = 48 - 16x$$

$$15x = 43$$

$$x = \frac{43}{15}$$

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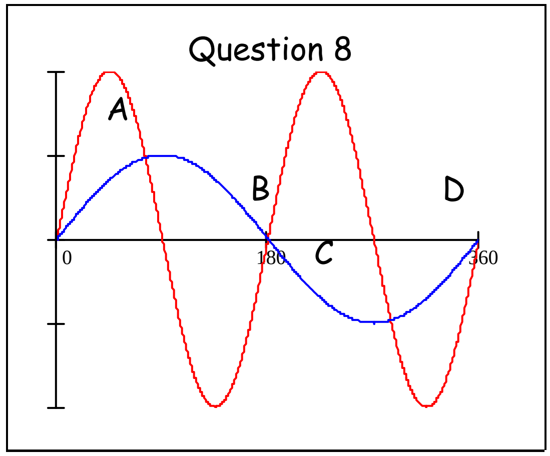


8. Given Sketch

$f(x) = k\sin(2x)$ $k > 1$

$g(x) = \sin(x)$

For points of intersection we have

$k \cdot \sin(2x) = \sin(x)$

$k \cdot 2\sin(x) \cdot \cos(x) = \sin(x)$

Since x is not 0 at A or C (from the diagram) we have

$$\frac{k \cdot 2 \cdot \cos(x)}{\sin(x)} = \frac{\sin(x)}{\sin(x)}$$ $$k \cdot 2 \cdot \cos(x) = 1$$

$$\cos(x) = \frac{1}{2k}$$

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9. Given information and equation

$$V = 252 \cdot e^{-0.06335t}$$

(a) The initial value ( at launch ) is when t = 0.

$$V = 252 \cdot e^{-0.06335 \times 0} = 252 \cdot 1 = £252\text{million}$$

(b) When V = 20 million we have

$$20 = 252 \cdot e^{-0.06335t}$$

$$\frac{20}{252} = e^{-0.06335 \cdot t}$$

$$\log_e\left(\frac{20}{252}\right) = -0.06335t$$

$$t = \frac{\log_e\left(\frac{20}{252}\right)}{-0.06335}$$

$$t = 40\text{years}$$

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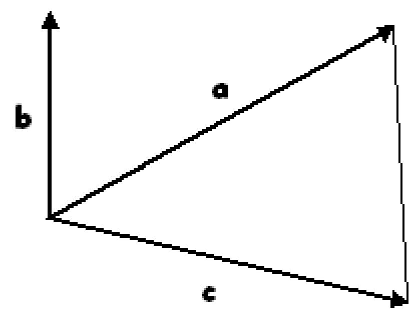
10. Given diagram and information




Keywords equilateral triangle
b perpendicular to a and c

$a = c = 3$units $b = 2$units

Evaluating $a \cdot (a + b + c)$

we get $a \cdot a = 3 \cdot 3 = 9$ $a \cdot b = 0$ $a \cdot c = 3 \cdot 3 \cdot \cos(60) = 9 \cdot \frac{1}{2} = 4.5$

$a \cdot (a + b + c) = 9 + 0 + 4.5 = 13.5$

11. (a) Given x = -1 then we have

$x^3 + px^2 + px + 1 = 0$

$(-1)^3 + p(-1)^2 + p(-1) + 1 = 0$

$-1 + p - p + 1 = 0$

$0 = 0$

OR

By synthetic division

| | 1 | p | p | 1 |
|---|---|---|---|---|
| -1 | | -1 | 1-p | -1 |
| | 1 | p-1 | 1 | 0 |

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11. (b) From synthetic division above we can write

$$x^3 + px^2 + px + 1 = (x+1)\left[x^2 + (p-1)x + 1\right]$$

Using the discriminant we have

$a = 1$ $\quad b = (p-1)$ $\quad c = 1$

For real roots we must have

$$b^2 - 4ac \geq 0$$

$$(p-1)^2 - 4\cdot1\cdot1 \geq 0$$

$$p^2 - 2p + 1 - 4 \geq 0$$

$$p^2 - 2p - 3 \geq 0$$

$$(p-3)(p+1) \geq 0$$

**Hence p must be in the range**

$p \leq 1$ and $p \geq 3$