

Higher Still Level Paper 1 2004

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1. Given the point A(7,4) and that the equations

$$x + 3y + 1 = 0 \text{ and } 2x + 5y = 0$$

intersect at the point B. We have :-

(a) Gradient of AB is

Intersection point B is given by solving by the simultaneous equations.

$$\begin{aligned} x + 3y + 1 &= 0 \\ 2x + 5y &= 0 \end{aligned}$$

$x = 5$ and $y = -2$ Hence B has coordinates (5,-2)

Hence Gradient of AB is $m_{AB} = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(-2 - 4)}{(5 - 7)} = 3$

(b) To show AB is perpendicular to one line we have :-

If perpendicular then the following is true

$$m_1 \cdot m_2 = -1$$

Equation 1 we have $x + 3y + 1 = 0 \Rightarrow y = \frac{-1}{3} \cdot x - \frac{1}{3}$

$m_1 \cdot m_{AB} = \frac{-1}{3} \cdot 3 = -1$ Hence AB and equation 1 are perpendicular.

Equation 2 we have $2x + 5y = 0 \Rightarrow y = \frac{-2}{5} \cdot x$

$m_2 \cdot m_{AB} = \frac{-2}{5} \cdot 3 \neq -1$ Hence AB and equation 2 are NOT perpendicular.

Hence only 1 equation is perpendicular with AB.

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2. **Given:** $f(x) = x^3 - x^2 - 5x - 3$

(a) If $(x+1)$ is a factor of $f(x)$ then remainder will be 0.

Using synthetic division we have.

$$\frac{x^3 - x^2 - 5x - 3}{(x + 1)} = 3x^2 - x - 5 \quad (\text{no remainder})$$

(a)(ii) Factorising $f(x)$ further we get:

$$f(x) = x^3 - x^2 - 5x - 3 = (x + 1)(3x^2 - x - 5)$$

$$3x^2 - x - 5 = (x + 1)^2(x - 3)$$

Hence we have

$$f(x) = x^3 - x^2 - 5x - 3 = (x + 1)^2(x - 3)$$

(b) Turning points are given by $f'(x) = 0$ we have:

$$\frac{d}{dx}f(x) = 3x^2 - 2x - 5 = 0 \quad 3x^2 - 2x - 5 = (3x - 5)(x + 1) = 0$$

$$x = -1 \text{ and } x = \frac{5}{3}$$

$$x = -1 \quad f(x) = (-1)^3 - (-1)^2 - 5(-1) - 3 = 0 \quad (-1, 0)$$

Since we are told that only one turning point lies on the x-axis it must be $(-1, 0)$

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3. **Given:** $\tan^2(x) = 3$ $0 \leq x \leq 2\pi$

Solving we get

$\tan^2(x) = 3$ **Note :** $\sqrt{3} \cdot \sqrt{3} = 3$ and $-\sqrt{3} \cdot -\sqrt{3} = 3$

$\tan x = \sqrt{3}$ and $-\sqrt{3}$

Hence

$\tan x = \sqrt{3}$ $\tan x = -\sqrt{3}$

$x = 60^0$ and $x = 240^0$ $x = 120^0$ and $x = 300^0$

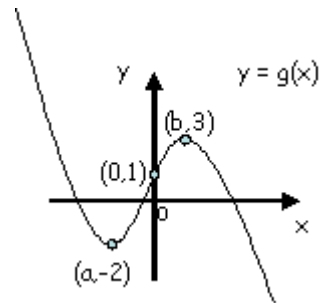
Hence overall solution is $x = 60^0, 120^0, 240^0, 300^0$

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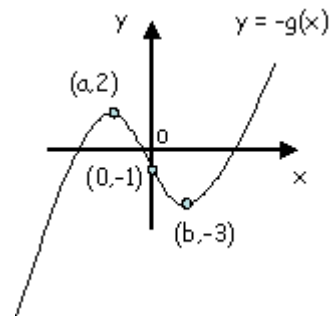
4. Given the sketch:

$$y = g(x)$$



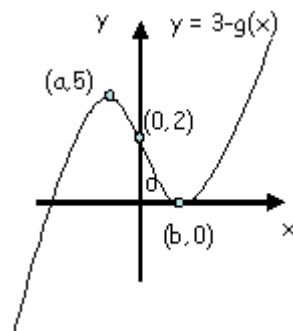
(a) To sketch $y = -g(x)$

Reflect in the x axis.



(b) To sketch $y = 3 - g(x)$

We simply move the $y = -g(x)$ 3 units in the positive y axis direction.



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5. Given $A(-3,4,7)$, $B(-1,8,3)$, $C(0,10,1)$

(a) If collinear then the following will be true:-

$$\vec{AB} = k \cdot \vec{AC}$$

$$\vec{AB} = \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} \quad \vec{AC} = \begin{pmatrix} 0 \\ 10 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = \frac{2}{3} \cdot \begin{pmatrix} 3 \\ 6 \\ -6 \end{pmatrix}$$

Hence points are collinear since $\vec{AB} = \frac{2}{3} \cdot \vec{AC}$

(b) To find D such that $\vec{AD} = 4 \cdot \vec{AB}$

We have

$$\vec{AD} = 4 \cdot \vec{AB}$$

$$d - a = 4(b - a)$$

$$d = 4b - 3a$$

$$d = 4 \cdot \begin{pmatrix} -1 \\ 8 \\ 3 \end{pmatrix} - 3 \cdot \begin{pmatrix} -3 \\ 4 \\ 7 \end{pmatrix} = \begin{pmatrix} -4 \\ 32 \\ 12 \end{pmatrix} - \begin{pmatrix} -9 \\ -12 \\ 21 \end{pmatrix} = \begin{pmatrix} 5 \\ 20 \\ -9 \end{pmatrix}$$

$$D = (5, 20, -9)$$

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6. Given: $y = 3 \sin(x) + \cos(2x)$

To find $\frac{d}{dx}y$ we use the chain rule.

$$\frac{d}{dx}y = 3 \cos(x) - 2 \cdot \sin(2x)$$

7. Given: $\int_0^2 \sqrt{4x+1} \, dx$

We have

$$\int_0^2 \sqrt{4x+1} \, dx = \frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2} \cdot 4} = \left[\frac{(4 \cdot 2 + 1)^{\frac{3}{2}}}{\frac{3}{2} \cdot 4} \right] - \left[\frac{(4 \cdot 0 + 1)^{\frac{3}{2}}}{\frac{3}{2} \cdot 4} \right]$$

$$\frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \frac{13}{3}$$

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8. Given: $x^2 - 10x + 27$

(a) To put in the form $(x + b)^2 + c$

We have using the completing the square method:-

$$x^2 - 10x + 27 = (x - 5)^2 + 27 - 25 = (x - 5)^2 + 2$$

(b) To show a function is always increasing we have to show $f'(x)$ is always greater than 0.

$$g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$$

$$\frac{d}{dx}g(x) = x^2 - 10x + 27 = (x - 5)^2 + 2$$

We can see that for any value of x the term

$$(x - 5)^2 > 0 \quad \text{Also} \quad 2 > 0$$

$$\text{Hence } (x - 5)^2 + 2 > 0$$

This result tells us that the function is always increasing.

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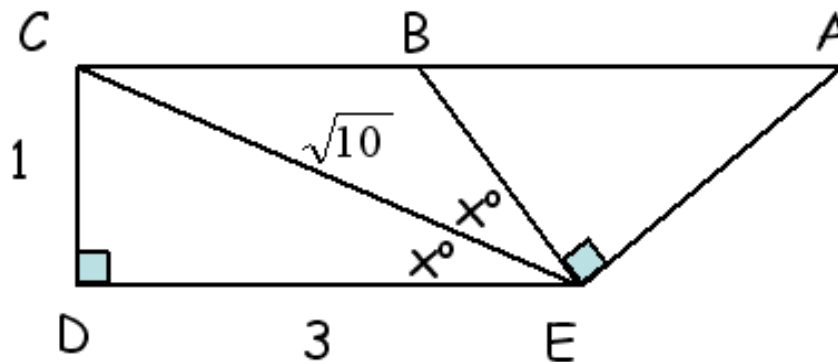
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9. **Given:** $\log_2(x + 1) - 2\log_2(3) = 3$

To find x we use the rules for logs and powers.

$$\log_2\left[\frac{(x+1)}{9}\right] = 3 \qquad 2^{\log_2\left[\frac{(x+1)}{9}\right]} = 2^3 \qquad \frac{x+1}{9} = 8 \qquad x = 72 - 1 = 71$$

10. **Given** the diagram in the question we can deduce the following:



The exact value of $\cos(\text{DEA})$ is

$$\cos(\text{DEA}) = \cos(2x + 90) = \cos(2x) \cdot \cos(90) - \sin(2x) \cdot \sin(90)$$

This reduces to $-\sin(2x)$ Since $\cos(90) = 0$ $\sin(90) = 1$

Also $-\sin(2x) = -2\sin(x) \cdot \cos(x)$ $\sin(x) = \frac{1}{\sqrt{10}}$ $\cos\left(\frac{3}{\sqrt{10}}\right)$

Hence

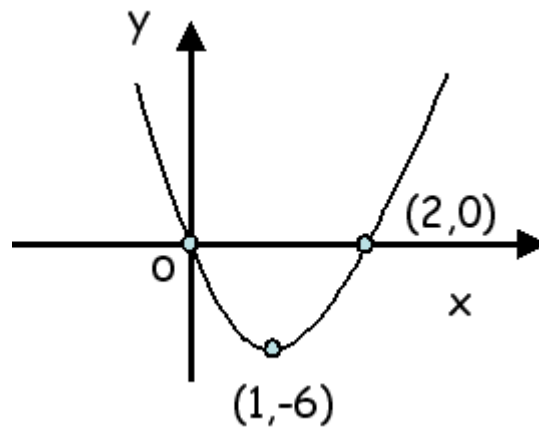
$$\cos(\text{DEA}) = -2 \cdot \sin(x) \cdot \cos(x) = -2 \cdot \frac{1}{\sqrt{10}} \cdot \frac{3}{\sqrt{10}} = \frac{-6}{10} = \frac{-3}{5}$$

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11. Given the diagram and the equation:

$$y = a \cdot x(x - b)$$



(a) To find a and b we have $(2, 0)$

$$0 = a \cdot 2 \cdot (2 - b) = 4a - 2 \cdot a \cdot b \quad 0 = 4 \cdot a - 2 \cdot a \cdot b \quad -4 \cdot a = -2 \cdot a \cdot b \quad b = \frac{-4}{-2} = 2$$

Substituting $b = 2$ for the equation at $(1, -6)$ gives

$$-6 = a \cdot 1 \cdot (1 - 2) = a - 2 \cdot a \quad -6 = -a \quad a = 6$$

Hence we have $a = 6$ $b = 2$ $y = 6x^2 - 12x$

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(b) Given that the equation in part (a) is

$$y = \frac{d}{dx}f(x) \quad \text{and} \quad f(1) = 4$$

To find $f(x)$ we integrate

$$\int \frac{d}{dx}f(x) \, dx = \int (6x^2 - 12x) \, dx = 2 \cdot x^3 - 6 \cdot x^2 + C$$

Find C we have

$$f(1) = 4 \quad 2 \cdot 1^3 - 6 \cdot 1^2 + C = 4 \quad C = 4 + 6 - 2 = 8$$

Hence

$$f(x) = 2 \cdot x^3 - 6 \cdot x^2 + 8$$