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Q1. Given $4x + y - 1 = 0$ and (-1,3).

$4x + y - 1 = 0$

$y = -4x + 1$

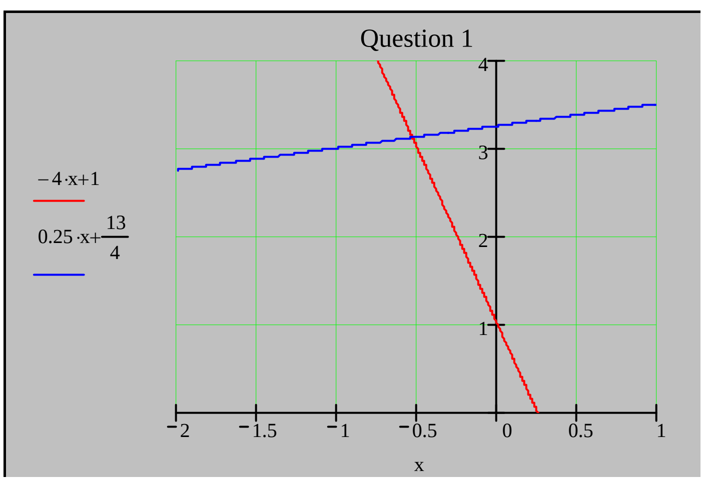
gradient perpendicular to $y = -4x + 1$ is

$m_1 \cdot m_2 = -1$

$\Rightarrow -4 \cdot m_2 = -1 \Rightarrow m_2 - \frac{1}{4}$

Hence equation of line perpendicular to $y = -4x + 1$ and passing through (-1,3) is

$$y - 3 = \frac{1}{4}(x - (-1)) \Rightarrow y - 3 = \frac{1}{4}x + \frac{1}{4} \Rightarrow x - 4y + 13 = 0$$



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Q2. Given $f(x) = x^2 + 6x + 11$

(a)

Completing the square we get

$(x+3)^2 + 2$: method $\frac{1}{2}$ the constant term of x; put in the form (x - 3)

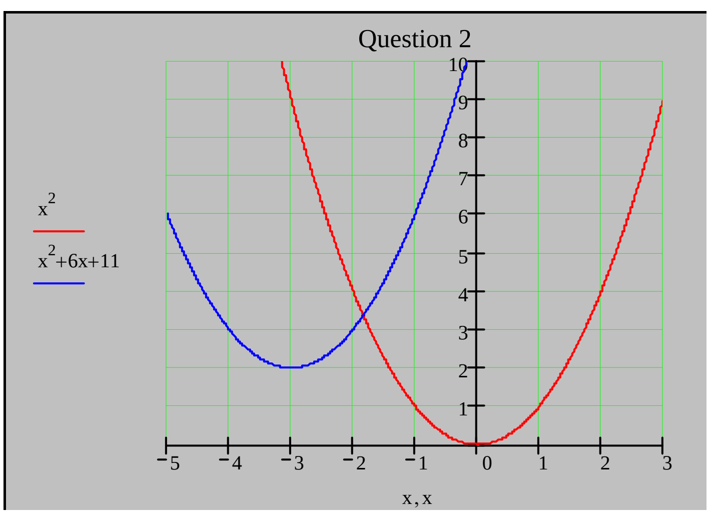
then mulitply out the brackets and add or subtract a

number (in this case 2) so that constant terms add up to 11

(b) Starting from the basic graph $y = x^2$

The term $(x+3)$ means move basic graph 3 units to the left

the constant 2 means move 2 units upwards



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Q3. $\underline{u} = 3\underline{i} + 2\underline{j}$ and $\underline{v} = 2i - 3\underline{j} + 4\underline{k}$

If u and v are perpendicular then

$\underline{u} \cdot \underline{v} = a_1b_1 + a_2b_2 + a_3b_3 = 0 \Rightarrow [(3 \cdot 2) + (2 \cdot (-3)) + (0 \cdot 4)] = 6 + (-6) + 0 = 0$

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Q4. Given $U_{n+1} = pU_n + q \quad -1 < p < 1$ and $U_0 = 12,\ U_1 = 15$ and $U_2 = 16$

(a) To find p and q we plug in the values $U_0 = 12,\ U_1 = 15$ and $U_2 = 16$ into the equation to get 2 simultaneous equations and then solve them.

$15 = 12p + q \quad -(1)$

$16 = 15p + q \quad -(2)$

Subtracting (1) from (2) we get $1 = 3p \quad p = \frac{1}{3}$

sub in $p = \frac{1}{3}$ into equation (1) or (2) to get q

$$15 = 12 \cdot \frac{1}{3} + q \quad \Rightarrow q = 15 - 4 = 11$$

(b) To find the limit, let L be the limit, we have

$$L = pL + q \quad \Rightarrow L = [\frac{q}{(1-p)}] = \frac{11}{(1 - \frac{1}{3})} = \frac{33}{2}$$

Q5. Given $f(x) = \sqrt{x} + \frac{2}{x^2}$

differentiating we get $f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} - 4 \cdot x^{-3}$

Hence $f'(4) = \frac{1}{2} \cdot (4)^{-\frac{1}{2}} - 4 \cdot (4)^{-3} = \frac{1}{4} - \frac{4}{64} = \frac{1}{4} - \frac{1}{16} = \frac{4}{16} - \frac{1}{16} = \frac{3}{16}$

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Q5. Given $f(x) = \sqrt{x} + \frac{2}{x^2}$

differentiating we get $f'(x) = \frac{1}{2} \cdot x^{-\frac{1}{2}} - 4 \cdot x^{-3}$

Hence $f'(4) = \frac{1}{2} \cdot (4)^{-\frac{1}{2}} - 4 \cdot (4)^{-3} = \frac{1}{4} - \frac{4}{64} = \frac{1}{4} - \frac{1}{16} = \frac{4}{16} - \frac{1}{16} = \frac{3}{16}$

Q6. Given A (-1,-3,2) and B (2,-1,1) and AB = BC = CD

(a) To find D we have

$$3AB = 3 \cdot \left[\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} - \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}\right] = \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix} \quad \Rightarrow \text{D is equal to } A + \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix} + \begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -1 \end{bmatrix}$$

D has cordinates (8,3,-1)

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Q7. Given $y = 2x + 1$ and $y = x^2 + 3x + 4$

If line intersects parabola then $b^2 - 4ac > 0$ for

$2x + 1 = x^2 + 3x + 4 \quad \Rightarrow x^2 + x + 3 = 0 \Rightarrow a = 1 \;\; b = 1 \;\; c = 3$

Hence $\quad b^2 - 4ac = 1^2 - 4 \cdot (1) \cdot 3 = 1 - 12 < 0$

Therefore line does not intersect parabola

Q8. Given $\displaystyle\int_0^1 \frac{dx}{(3x+1)^{1/2}}$

Using the formula $\displaystyle\int (ax+1)^n \, dx = \frac{(ax+1)^{n+1}}{a(n+1)} + C$

Hence

$$\int_0^1 \frac{dx}{(3x+1)^{1/2}} = \int (3x+1)^{-\frac{1}{2}} = \left[\frac{(3x+1)^{\frac{1}{2}}}{3 \cdot \frac{1}{2}} \right]_0^1 = \frac{2}{3}\left[(3 \cdot 1 + 1)^{\frac{1}{2}} - (3 \cdot 0 + 1)^{\frac{1}{2}} \right] = \frac{2}{3}$$

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Q9. Given $f(x) = \frac{1}{x - 4}$ and $g(x) = 2x + 3$

(a) $h(x) = f(g(x)) = \frac{1}{(2x+3) - 4} = \frac{1}{2x - 1}$

(b) Restriction is that the denominator cannot be 0 hence

$2x - 1 = 0 \quad x = \frac{1}{2} \quad \text{domain is } x \in R - \{0\}$

Q10. Given A (8,4) and OA is incline at p to the x - axis

(a) (i)

$$\sin(2p) = 2 \cdot \sin(p) \cdot \cos(p) = 2 \cdot \frac{4}{\sqrt{80}} \cdot \frac{8}{\sqrt{80}} = \frac{64}{80} = \frac{8}{10} = \frac{4}{5}$$

(ii)

$$\cos(2p) = \cos^2(p) - \sin^2(p) = \frac{64}{80} - \frac{16}{80} = \frac{48}{80} = \frac{3}{5}$$

(b) Given that OB is inclined at 2p to the x - axis

$$\text{gradient of } OB = \tan(2p) = \frac{\sin(2p)}{\cos(2p)} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

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Q11. Given O centre (0,0), A and B are centres of circles and O and B circles are congruent and also touch circle A. Also circle A has equation $(x-12)^2+(y+5)^2=25$

(a) (i) A has coordinates (12, -5) and the length OA is given by

$$OA=\sqrt{((12)^2+(-5)^2)}=\sqrt{169}=13$$

(ii) Since O and B are congruent then B has centre (24,0) and AB = 13 by part (a)(i)

Hence radius of Circle B is $r_B+r_A=13 \Rightarrow r_B=13-5=8$

Equation of circle B is therefore $(x-24)^2+y^2=(8)^2$

(b) Given the equation of the parabola that passes through the centres can be written in the form $y=px(x+q)$ we have

$-5=144p+12pq \quad *(1)$

$$0=24^2p+24pq \Rightarrow 0=576p+24pq \Rightarrow pq=\frac{-576p}{24}=-24p \; *(2)$$

Solving these simultaneous equations we get

$$-5=144p+12(-24p) \Rightarrow -5=144p-288p \Rightarrow -5=-144p \Rightarrow p=\frac{5}{144}$$

$$\text{and } pq=-24p \Rightarrow q=\frac{-24p}{p}=-24$$

Q12 . Given $3\log_e(2e)-2\log_e(3e)$ we can simplify using rules for logs to

$$\begin{aligned}3\log_e(2e)-2\log_e(3e) &= 3[\log_e(2)+\log_e(e)]-2[\log_e(3)+\log_e(e)]\\ &= 3[\log_e(2)+1]-2[\log_e(3)+1]\\ &= 3\log_e(2)+3-2og_e(3)-2\\ &= 3\log_e(2)-2\log_e(3)+1\\ &= \log_e(2)^3-\log_e(3)^2+1\\ &= 1+\log_e(8)-\log_e(9)\end{aligned}$$

Hence A = 1 B = 8 C = 9