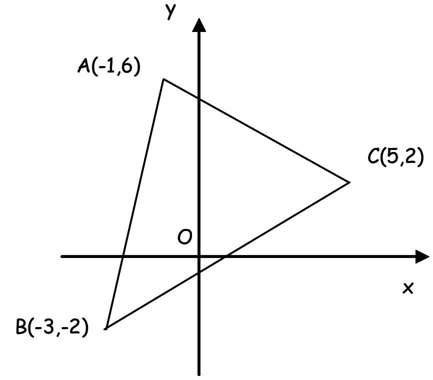
Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

1. Given the diagram opposite we have



(a) Mid-point from AB is

$$\left[\frac{[-1+(-3)]}{2}, \frac{[6+(-2)]}{2}\right] = (-2, 2)$$

Equation of the median line p from C is y = 2

(b) Gradient of BC

$$m_{bc} = \frac{[2-(-2)]}{[5-(-3)]} = \frac{4}{8} = \frac{1}{2}$$

Gradient of bisector is -2 since

$$m_{bc} \cdot m_{bisector} = -1$$

Midpoint of BC is

$$\left[\frac{[5+(-3)]}{2}, \frac{[2+(-2)]}{2}\right] = (-1, 0)$$

Equation of bisector q is

$$y - 0 = -2(x - 1)$$
$$2x + y = 2$$

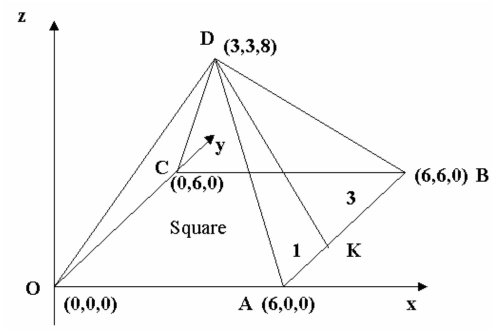
(c) Intersection co-ordinates of p and q are

$-2x + 2 = 2$ x=0 Co-ordinates are (0, 2)

Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

2. Given A (6, 0, 0) and D (3, 3, 8)



(a) C is (0, 6, and 0)

(b) $\overrightarrow{DA} = \overrightarrow{DC} + \overrightarrow{CO} + \overrightarrow{OA} = (-3,3,-8) + (0,-6,0) + (6,0,0) = (3,-3,-8)$

$\overrightarrow{DB} = (6,6,0) - (3,3,8) = (3,3,-8)$

(c) Angle ADB

Using the dot product rule we have

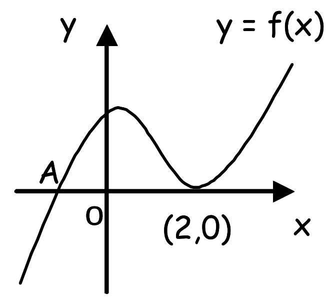
$$\cos(ADB) = \frac{\overrightarrow{DA}\cdot\overrightarrow{DB}}{|DA|\cdot|DB|} = \frac{(3,-3,-8)\cdot(3,3,-8)}{\sqrt{(9+9+64)}\cdot\sqrt{(9+9+64)}}$$

$$\cos(ADB) = \frac{64}{82} = \frac{32}{41}$$

$$\text{angle}(ADB) = \cos^{-1}\left(\frac{32}{41}\right) = 38.7^o$$

Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA



3. Given $y = 2x^3 - 7x^2 + 4x + 4$

(a) Max / Mini is given by

$$\frac{d}{dx}y = 0$$

$$\frac{d}{dx}y = 6x^2 - 14x + 4 = 0$$

This factorises to give

$$(3x - 1)(2x - 4) = 0$$

$x = 2$ and $x = \frac{1}{3}$

From the sketch it is clear that

$x = 2$ is a minimum and $x = \frac{1}{3}$ is a maximum

(b) From the sketch clearly (x-2) is a factor.
Using synthetic division we get

$$2x^3 - 7x^2 + 4x + 4 = (x - 2)\left(2x^2 - 3x - 2\right)$$
$$2x^2 - 3x - 2 = (2x + 1)(x - 2)$$

Hence we have $(2x + 1)(x - 2)^2$

Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

(c) The point A is $\left(\frac{-1}{2}, 0\right)$: x from part (b) 2x-1 = 0

Hence

$$2x^3 - 7x^2 + 4x + 4 < 0 \quad \text{for} \quad x < \frac{-1}{2}$$

4. From Information given we can write:-

Using the recurrence formula $U_{n+1} = a \cdot U_n + C$

We have

$$U_{n+1} = U_n + 0.5 - 0.2 \cdot U_n$$

$$U_{n+1} = 0.8 \cdot U_n + 0.5$$

(a) Limiting value is found by

$$L = 0.8 \cdot L + 0.5$$

rearranging we get $L = \frac{0.5}{(1-0.8)} = 2.5 \text{metres}$

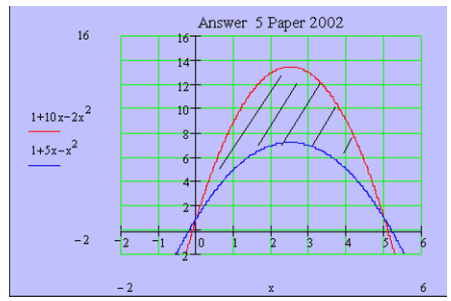
(b) To make sure they do not grow more than 2 metres we need:-

$L = \frac{0.5}{(1-a)} = 2$ rearranging we get $a = 0.75$

Hence we need to trim the hedges by at least 25 % each year.

Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA



5. Given the sketch and the equations:-

$y = 1 + 10x - 2x^2 \qquad y = 1 + 5x - x^2$

(a) To get the points of intersection we have

$$1 + 10 \cdot x - 2 \cdot x^2 = 1 + 5 \cdot x - x^2$$

$$x^2 - 5x = 0$$

$$x \cdot (x - 5) = 0$$

$$x = 0 \quad \text{and} \quad x = 5$$

(b) To find shaded area we have:-

$$A = \int_0^5 \left(1 + 10 \cdot x - 2 \cdot x^2\right) - \left(1 + 5 \cdot x - x^2\right) dx$$

$$A = \int_0^5 \left(-x^2 - 5 \cdot x\right) dx = \left(-\frac{x^3}{3} - 5 \cdot \frac{x^2}{2}\right) = \left(-\frac{5^3}{3} - 5 \cdot \frac{5^2}{2}\right) - \left(-\frac{0^3}{3} - 5 \cdot \frac{0^2}{2}\right)$$

$$A = \frac{125}{6}$$

Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

6. Given:- $y = 2\cdot\sin\left(x - \frac{\pi}{6}\right)$ $\quad x = \frac{\pi}{3}$

For $x = \frac{\pi}{3}$

$$y = 2\cdot\sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = 2\cdot\sin\left(\frac{\pi}{6}\right) = 1$$

Hence co-ordinates P is $\left(\frac{\pi}{3}, 1\right)$

Value of gradient at $x = \frac{\pi}{3}$ is given by $\frac{d}{dx}y\left(\frac{\pi}{3}\right)$

Using the chain rule we get

$$\frac{d}{dx}y = 2\cdot\cos\left(x - \frac{\pi}{6}\right)$$

$$\frac{d}{dx}y\left(\frac{\pi}{3}\right) = 2\cdot\cos\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = 2\cdot\cos\left(\frac{\pi}{6}\right) = \sqrt{3}$$

We have $\text{gradient} = \sqrt{3}$

$$P\left(\frac{\pi}{3}, 1\right)$$

Hence equation of the tangent is

$$y - 1 = \sqrt{3}\cdot\left(x - \frac{\pi}{3}\right) \qquad y = \sqrt{3}\cdot x + \left(1 - \frac{\pi}{\sqrt{3}}\right)$$

Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

7. Given: - $y = \log_3(x-2) + 1$ $y = 0$ (the x axis)

They intersect when

$$\log_3(x-2) + 1 = 0$$

Using the rules of logs we take antilog of each side.

$$\log_3(x-2) + 1 = 0 \qquad \log_3(x-2) = -1 \qquad (x-2) = 3^{-1} \qquad x = 2 + \frac{1}{3} = \frac{7}{3}$$

8. Given: - $a = 2\cdot(4-t)^{\frac{1}{2}}$ and the point starts from REST.
To get the velocity equation we integrate the acceleration equation above.

$$v = \int a\,dt = \int 2\cdot(4-t)^{\frac{1}{2}}\,dt = 2\cdot\frac{2}{3}\cdot(4-t)^{\frac{3}{2}}\cdot(-1) + C$$

$$v = \frac{-4}{3}\cdot(4-t)^{\frac{3}{2}} + C$$

when t = 0 then v = 0 (starting from rest) then we have

$$0 = \frac{-4}{3}\cdot(4-0)^{\frac{3}{2}} + C \qquad C = \frac{32}{3}$$

Hence equation is $v = \frac{-4}{3}\cdot(4-t)^{\frac{3}{2}} + \frac{32}{3}$

Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

9. Given:- $(1 - 2k) \cdot x^2 - 5 \cdot kx - 2 \cdot k = 0$

To have real roots for all integer values of k we need

$$b^2 - 4ac \geq 0$$

Hence we have

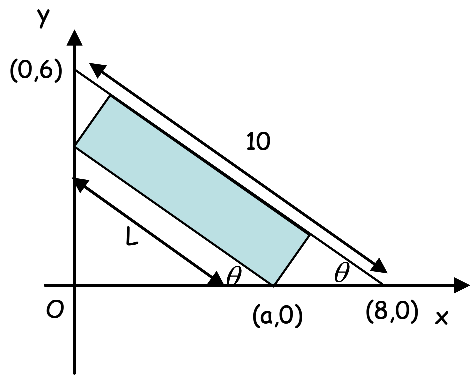
$$(-5k)^2 - 4 \cdot (1 - 2k)(-2k) \geq 0$$
$$25k^2 + 8k - 16k^2 \geq 0$$
$$9k^2 + 8k \geq 0$$

Since $9k^2$ is always positive for all values of k and $9k^2 > 8k$ for all integers values of k, then the equation has real roots for all integer values of k.

Higher Still Level Exam 2002 Paper 2

Created by
Graduate Bsc (Hons) MathsSci (Open) GIMA

10. Given the sketch:-



(a)(i) $\cos(\theta) = \frac{8}{10} = \frac{a}{L}$

Rearranging we get $L = \frac{5}{4} \cdot a$

(b)(ii) $\sin(\theta) = \frac{b}{(8-a)} = \frac{6}{10}$

Rearranging we get $b = \frac{3}{5} \cdot (8-a)$

$$\text{Area}_{\text{rectangle}} = L \cdot b = \frac{5}{4} \cdot a \cdot \left[\frac{3}{5} \cdot (8-a)\right] = \frac{3}{4} \cdot a \cdot (8-a)$$

Maximum / Minimum values are when A'(a) = 0.

$$A = \frac{3}{4} \cdot a \cdot (8-a) = 6a - \frac{3}{4} \cdot a^2$$

$$\frac{d}{da} A = 6 - \frac{6 \cdot a}{4} = 0$$

rearranging we get

$$a = \frac{24}{6} = 4$$

This is a maximum value because the constant term for a^2 in the equation A (a) is negative (-3/4).