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A1. Given $(x+1)^2 + (y-1)^2$ and the point P(2,3) on the circle.

Circles centre is $(-g,-f) = (-1,1)$

Gradient of P to centre is $m_{PC} = \dfrac{(3-1)}{(2-(-1))} = \dfrac{2}{3}$

Gradient of perpendicular is $m_p = \dfrac{-3}{2}$; since $m_p \cdot m_{PC} = -1$

Equation of line is therefore

$$y - b = m(x - a) \quad \text{where } a = 2 \quad b = 3$$

$$y - 3 = \frac{-3}{2}(x - 2)$$

$$3x + 2y = 12$$

A2. Given P(-1,-1,0), R(5,2,-3) and ratio 2 : 1

We have

$$\overrightarrow{PQ} = \frac{m}{(m+n)} \cdot \overrightarrow{PR} = \frac{2}{(2+1)} \cdot [(5,2,-3) - (-1,-1,0)] = (4,2,-2)$$

Q is

$$P + (4,2,-2) = (-1,-1,0) + (4,2,-2) = (3,1,-2)$$

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A3. Given $f(x) = \sin(x),\ g(x) = 2x$

Then

(i) $f(g(x)) = \sin(2x)$; x in f(x) is replaced by 2x

(ii) $g(f(x)) = 2 \cdot \sin x$; x in g(x) is replaced by sinx

(b) $2f(g(x)) = g(f(x))$

$2 \cdot \sin 2x = 2 \cdot \sin x$

$2 \cdot 2 \cdot \sin x \cos x = 2\sin x$

$2 \cdot \sin x(2\cos x - 1) = 0$

For $2 \cdot \sin x = 0$; $x = 0^o, 180^o, 360^0$

For $2\cos x - 1 = 0$; $x = 60^o$ and 300^o

Hence solution is $x = 0^o, 60^o, 180^o, 300^o, 360^o$ between 0^o and 360^o

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A4. Given $y = 2x^2 - 7x + 10$

Since tangent makes angle of 45° then gradient m is equal 1.

To find x coordinate we find

$y' = 4x - 7 = 1 \quad ; \; x = 2$

To find y coordinate sub x into original equation.

$y(2) = 2 \cdot (2)^2 - 7 \cdot 2 + 10 = 4$

Coordinates are (2,4)

A5. Given diagram

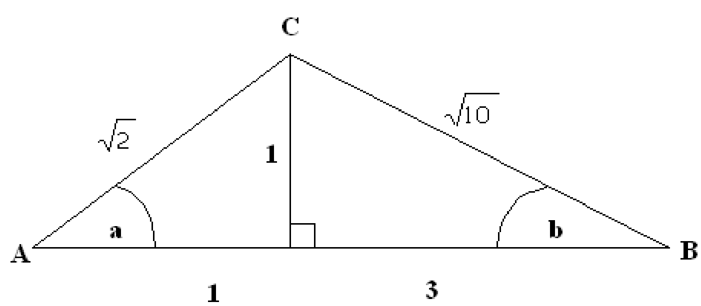
We have using trig formula

$$\sin(a+b) = \sin(a)\cdot\cos(b) + \cos(a)\cdot\sin(b)$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{10}}$$

$$= \frac{3}{2\sqrt{5}} + \frac{1}{2\sqrt{5}} = \frac{4}{2\sqrt{5}}$$

$$= \frac{2}{\sqrt{5}}$$



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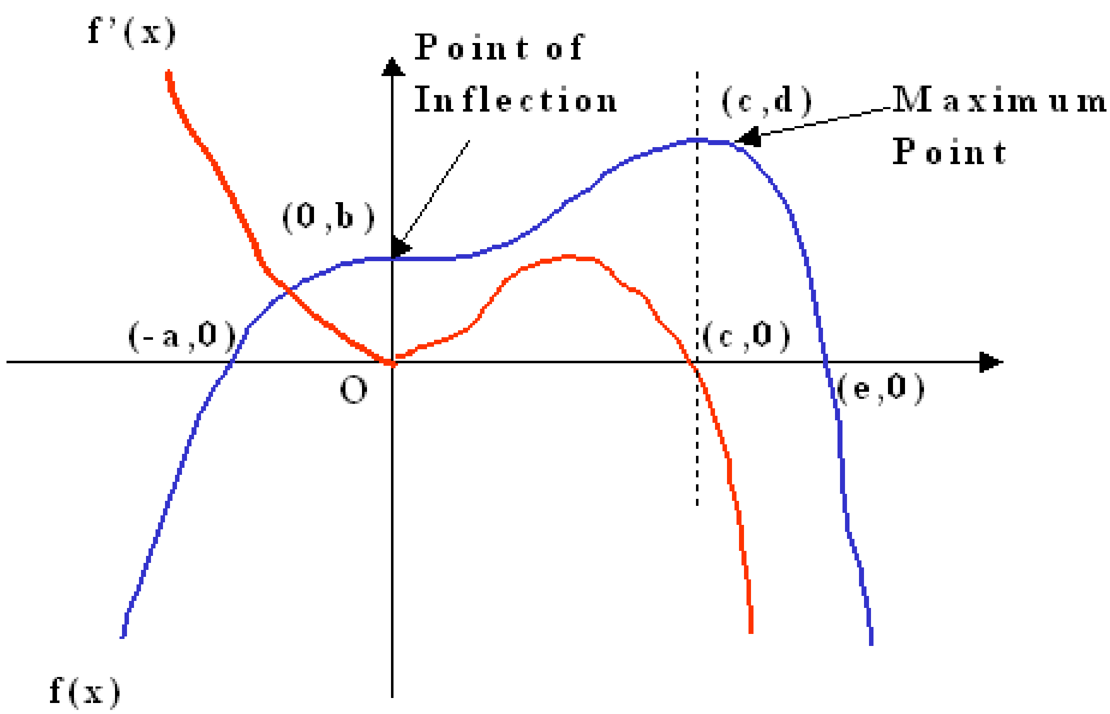
A6. See sketch below.

Note that points of inflection and Max / Mini points always have zero gradients. Going along the path of f(x) from left to right we can see the gradient is positive and decreasing. At (0, b) it is zero. After (0, b) it is positive and increasing then positive and decreasing until it reaches (c, d) when it is zero. After (c, d) it is negative and increasing.



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A7. Given $f(x) = x^2 - 4x + 5$

putting in the form $(x-a)^2 + b$ using the completing the squares method we get

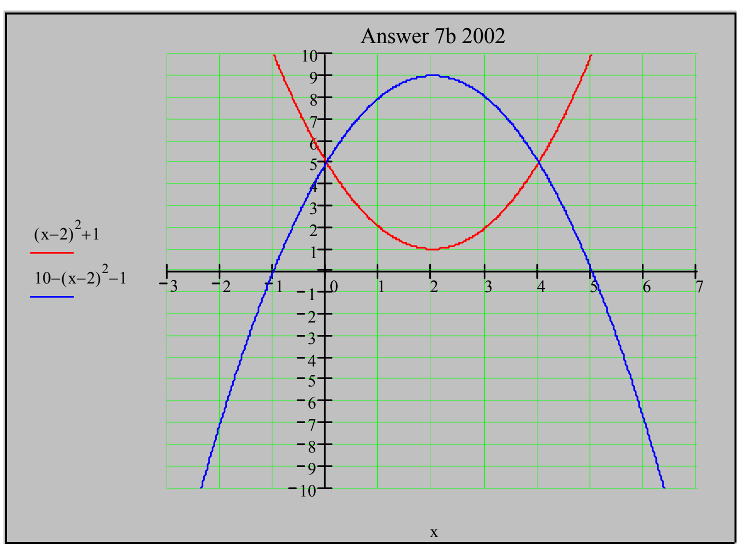
$x^2 - 4x + 5$

$(x-2)^2 + 5 - 4$

$(x-2)^2 + 1$

(b) (i) To sketch the graph f(x) we take the standard graph $y = x^2$ and move it 1 unit to the right (x - 2) and then 1 unit upwards (+1).
(ii) To sketch the graph y = 10 - f(x) we take f(x) and reflect it in the x - axis (-f(x)) and then move it 10 units upwards (+10).

(c) From the graph for 10 - f(x) to be postive we need $1 < x < 5$.



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A8.

(a) From the graph equation is $y = 2 \cdot \cos 2x$

2x ; since this cosine repeats itself every π

2 ; since the maxi/mini values are ± 2.

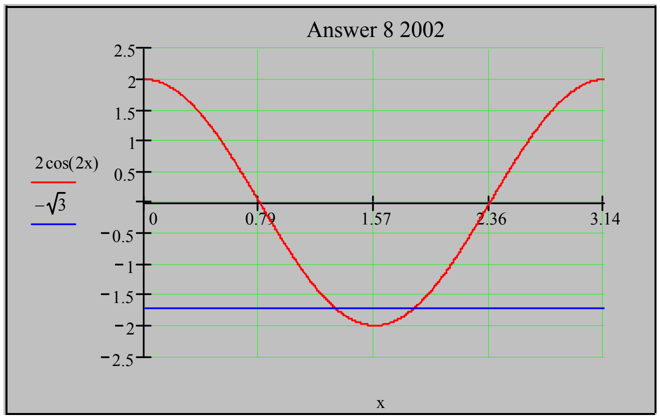
(b) Points of intersection are given by

$$2\cos 2x = -\sqrt{3}$$

$$\cos 2x = \frac{-\sqrt{3}}{2}$$

$$2x = \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6} \text{ and } \frac{7\pi}{6}$$

$$x = \frac{5\pi}{12} \text{ and } \frac{7\pi}{12}$$



From graph $A = \frac{5\pi}{12}$ and $B = \frac{7\pi}{12}$

To find y corordinate of B simply $y = -\sqrt{3}$.

$$B\left(\frac{7\pi}{12}, -\sqrt{3}\right)$$

A9. Given $\sin(x) - \cos(x)$

We have using trig formula

$k\sin(x - a) = k\sin(x)\cdot\cos(a) - k\cos(x)\cdot\sin(a)$

comparing with original equation we have

$k\sin(a) = 1$ and $k\sin(a) = 1$

squaring each side and adding we get

$[k\cos(a)]^2 + [k\sin(a)]^2 = 1^2 + 1^2$

$k^2[\cos^2(a) + \sin^2(a)] = 2$

$k^2 = 2$; since $\cos^2(a) + \sin^2(a) = 1$

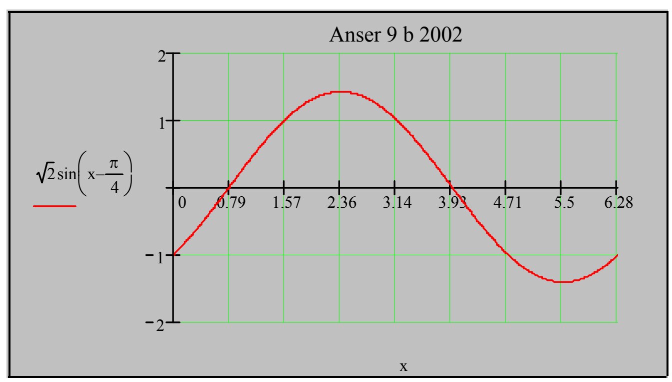
$k = \sqrt{2}$

dividing both sides we get

$$\frac{k\sin(a)}{k\cos(a)} = \tan(a) = \frac{1}{1}$$

$$a = \tan^{-1}(1) = \frac{\pi}{4}$$

Hence we have $\sqrt{2}\cdot\sin(x - \frac{\pi}{4})$



(b) See graph opposite

Max / mini values are $\pm\sqrt{2}$ at $x = \frac{\pi}{2}$ and π

Cuts x - axis at $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

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A10. Given $f(x) = (8 - x^3)^{1/2}$

Using the chain rule we get

$$f'(x) = \frac{1}{2} \cdot (8 - x^3)^{-1/2} \cdot (-3x^2) = \frac{-3x^2}{2(8 - x^3)^{1/2}}$$

(b) Given $\int \frac{x^2}{(8 - x^3)^{1/2}} dx$

From (a)above we get

$$\int \frac{x^2}{(8 - x^3)^{1/2}} dx = \frac{-2}{3}(8 - x^3)^{1/2} + c$$

This can be verified by finding $f'(x)$ of $f(x) = \frac{-2}{3}(8 - x^3)^{1/2}$

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A11. Given graph is of the form $y = kx^n$ A(0.5,0) B(0,1)

Taking $\log_5$ on both sides and applying log rules we get

$\log_5 y = \log_5 k + n\log_5 x$

$[\ \ Y\ =\ \ C\ \ + mX]$

From the graphic C i.e. $\log_5 k = 1$ hence $k = 5^1 = 5$

From the graphic gradient i.e. $n = \dfrac{(1-0)}{(0-0.5)} = -2$

Hence original equation is $y = 5x^{-2}$