Functions and Composites Examples

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1. For the given functions write down the domain.

(a) $p(x) = \dfrac{1}{(x+3)}$

(b) $p(x) = x^2 + x + 2$

(c) $p(x) = \dfrac{x+2}{\left(x^2 - 25\right)}$

(d) $p(x) = \sqrt{(8-x)}$

(e) $p(x) = \dfrac{1}{\sqrt{(x-2)}}$

Solution

The key is that you need to know that you cannot divide by zero and you cannot take the square root of a negative number.

(a) Cannot divide by zero hence:

$(x+3) = 0$
$x = -3$

Domain is x ε R - {-3}

(b) No restriction hence: Domain is x ε R

(c) Cannot divide by zero hence:

$\left(x^2 - 25\right) = (x+5)\cdot(x-5) = 0$
$x = 5$
$x = -5$

Domain is x ε R - {-5, 5}

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(d) Cannot get the square root of a negative number hence:

$(8 - x) \geq 0$ as along as $x \leq 8$ Domain is $x \leq 8$

(e) Cannot get the square root of a negative number and cannot divide by zero.

hence:

$(x - 2) \geq 0$ as along as $x \geq 2$

but cannot divide by zero

$x \neq 2$ Domain is $x > 2$

2. For p(x) = 2 xs+3 find the following:-

(a) $p(x^2)$

(b) $p(2x)$

(c) $p(2x^2)$

(d) $p(x^2 + x - 4)$

Solution

(a) $p(x^2) = 2\cdot(x^2) + 3 = 2\cdot x^2 + 3$

(b) $p(2\cdot x) = 2(2x) + 3 = 4\cdot x + 3$

(c) $p(2x^2) = 2\cdot(2\cdot x^2) + 3 = 4\cdot x^2 + 3$

(d) $p(x^2 + x - 4) = 2\cdot(x^2 + x - 4) + 3 = 2\cdot x^2 + 2\cdot x - 5$

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3. For $r(x) = x+1$ and $s(x) = x^2+3$ find the following:-

(a) $r(s(x))$

(b) $s(r(x))$

Solution

(a) $r(s(x)) = (x^2 + 3) + 1 = x^2 + 4$

(b) $s(r(x)) = (x + 1)^2 + 3 = x^2 + x + x + 1 + 3 = x^2 + 2x + 4$

4. For $r(y) = 2y$ and $s(y) = y^2 - k$, show that:-

$$(r(y))^2 - s(r(y)) = k$$

Solution

$(r(y))^2 = (2y)^2 = (2y^2) = 4y^2$

$s(r(y)) = (2y)^2 - k = 4y^2 - k$

Hence we have

$(r(y))^2 - s(r(y)) = 4y^2 - (4y^2 - k) = k$

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5. Find the function t(x) and its domain when t(x) = r(s(x)).

(a) $r(x) = 12x + 1 \qquad s(x) = \frac{1}{(x^2 - 9)}$

(b) $r(x) = 1 + 2x \qquad s(x) = \frac{1}{(x^2 - 1)}$

Simplify as much as possible.
(HINT: remember the rules of arithmetic for fractions!)

Solution

(a) $$t(x) = r(s(x)) = 12\left(\frac{1}{x^2 - 9}\right) + 1 = \frac{12}{x^2 - 9} + \frac{x^2 - 9}{x^2 - 9} = \frac{x^2 + 3}{x^2 - 9}$$

$$\frac{x^2 + 3}{x^2 - 9} = \frac{x^2 + 3}{(x + 3)\cdot(x - 3)}$$

Cannot divide by zero hence
$x \neq 3$
$x \neq -3$

Domain is x ϵ R - {-3, 3}

(b) $$t(x) = r(s(x)) = 1 - 2\left(\frac{1}{x^2 - 1}\right) = \frac{x^2 - 1}{x^2 - 1} - \frac{2}{x^2 - 1} = \frac{x^2 - 1}{x^2 - 1} - \frac{2}{x^2 - 1}$$

$$\frac{x^2 - 3}{x^2 - 1} = \frac{x^2 - 3}{(x + 1)\cdot(x - 1)}$$

Cannot divide by zero hence
$x \neq 1$
$x \neq -1$
Domain is x ϵ R - {-1, 1}

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6. Given the graph y = f(x). write down what effect the following operations have on the graph.

(a) $y = -f(x)$

(b) $y = f(x+1)$

(c) $y = f(x-3)$

(d) $y = -(-f(x))$

(e) $y = f(-x)$

(f) $y = f(x) - 2$

(g) $y = f(x) + 6$

(h) $y = -2 - f(x)$

(i) $y = -f(x-3)$

(j) $y = 4 + (-f(x+5))$

Solution

(a) Graph is reflected in the x-axis.

(b) Graph is moved 1 unit to the left.

(c) Graph is moved 3 units to the right.

(d) Graph is reflected twice in the x-axis which leaves the graph unchanged?

(e) Graph is reflected in the y-axis.

(f) Graph is moved 2 units downward.

(g) Graph is moved 6 units to the right.

(h) Graph is reflected in x-axis and then moved 2 units downward.
Combination of (a) and (f) above.

(i) Graph is moved 3 units to the right and then reflected in the x-axis.
Combination of (a) and (c) above.

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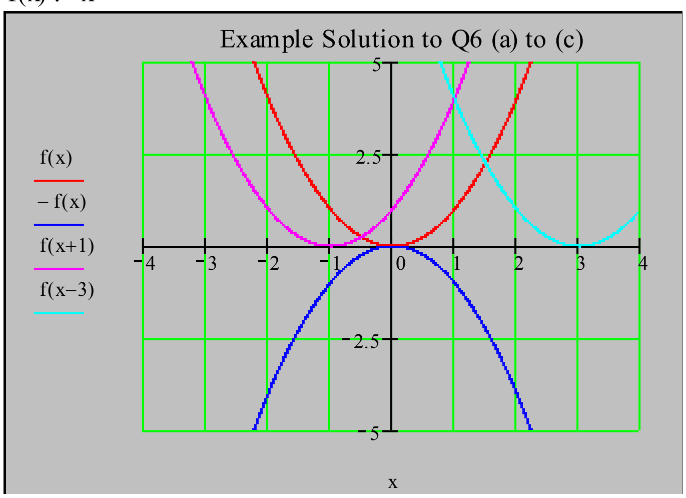
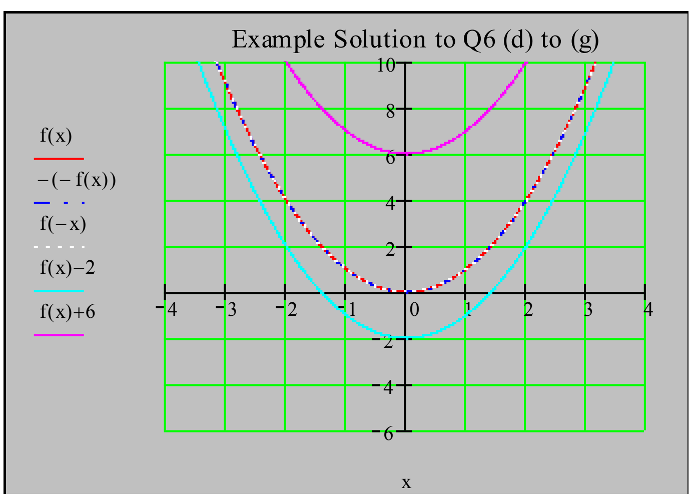
(j) Graph is moved 5 units to the left and then reflected in the x-axis and finally move graph 4 units upwards.

Examples of graphs for $y = f(x)$ are shown below:-

where

$f(x) := x^2$





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Notes

$f(x) = -(-f(x))$

Graphically

f(x) looks the same as f (-x) but they are **functionally** different