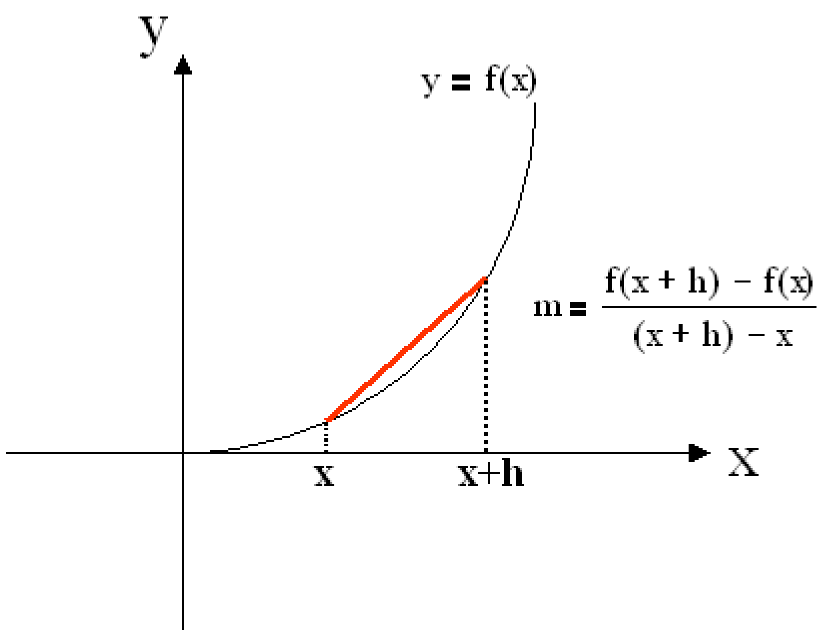
Differentiation KeyPoints

Created by Graduate Bsc (Hons) MathsSci (Open) GIMA

Differentiation is the process of finding the "rate of change" of a function. In the case of the straight line function the rate of change is constant and is known as the GRADIENT of the line.

To find an approximate "rate of change" for a function at a particular point x say, we draw a straight line between the point and a point very close to x, (x+h) say, and then calculate the gradient for the line. We can repeat this process for smaller and smaller values of h.



As h tends to 0 we have a better approximation to the "rate of change" at x. This leads us to the formal definition for differentiation:-

$$\frac{d}{dx}f = \lim_{h\to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

Differentiation KeyPoints

Created by Graduate Bsc (Hons) MathsSci (Open) GIMA

E.g. Differentiate the function below from first principles.

$$f(x) = x^2$$

Solution

$$\frac{d}{dx}f = \lim_{h\to 0} \frac{f(x+h) - f(x)}{(x+h) - x}$$

Using

$$\frac{d}{dx}f = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{(x+h) - x}$$

$$\frac{d}{dx}f = \lim_{h\to 0} \frac{\left(x^2 + 2\cdot x\cdot h + h^2\right) - x^2}{h}$$

$$\frac{d}{dx}f = \lim_{h\to 0} \frac{2\cdot x\cdot h + h^2}{h}$$

$$\frac{d}{dx}f = \lim_{h\to 0} \frac{2\cdot x - h}{1}$$

$$\frac{d}{dx}f = 2x$$

Differentiation KeyPoints

Created by Graduate Bsc (Hons) MathsSci (Open) GIMA

Fortunately we don't have to do everything from first principles we only have to remember some simple differentiation rules.

$$\frac{d}{dx}x^n = n\cdot x^{n-1} \qquad \frac{d}{dx}(ax+b)^n = an\cdot(ax+b)^{n-1}$$

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x$$

Chain rule

$$\frac{d}{dx}\left(x^k + 2\cdot x^s\right)^n = \frac{d}{dx}(\text{outside}\cdot\text{the}\cdot\text{bracket})\cdot\frac{d}{dx}(\text{inside}\cdot\text{the}\cdot\text{bracket})$$

1. If the differentiated function has a value of zero at a particular point then these points are called "Stationary Points" and can be a maximum, minimum or point of inflection.

$\frac{d}{dx}(y) = 0$ where y equals a function of

Differentiation KeyPoints

Created by Graduate Bsc (Hons) MathsSci (Open) GIMA

2. When you differentiate a cubic function you get a quadratic

$y = x^3 + x^2 + x + 1$ $\quad\quad \frac{d}{dx}y = 3 \cdot x^2 + 2 \cdot x + 1$

Differentiating a quadratic you get a straight line

$y = 3 \cdot x^2 + 2 \cdot x + 1$ $\quad\quad \frac{d}{dx}y = 6 \cdot x + 2$

Differentiating a straight line you get a constant.

$y = 6 \cdot x + 2$ $\quad\quad \frac{d}{dx}y = 6$

Differentiating a constant you get zero. (No rate of change!)

$y = 6$ $\quad\quad \frac{d}{dx}y = 0$

3. When we can find a value of the rate of change which is a maximum or minimum we can say that we have an optimizing condition (Stationary Point). This happens when

$$\frac{d}{dx}y = 0$$

4. Practical example of differentiating:-

If we have a function that gives the displacement of some object from some reference point, then differentiating the function gives us another function which defines the velocity of the object.

Differentiating the velocity function again, gives another function that defines the object's acceleration.