Circle Examples

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Graduate Bsc (Hons) MathsSci (Open) GIMA

1. Sketch the circles given by the following equations.
Comment on your results.

(a) $(x - 1)^2 + (y - 3)^2 = 16$

(b) $2x^2 + 2y^2 - 4x - 12y - 12 = 0$

Solution

(a) $(x - 1)^2 + (y - 3)^2 = 16$

Comparing with standard form

$(x - a)^2 + (y - b)^2 = r^2$

centre $= (a, b)$ radius $= r$

Hence we have

centre $= (1, 3)$ $r = \sqrt{16} = 4$

(b) $2x^2 + 2y^2 - 4x - 12y - 12 = 0$

comparing with standard form

$x^2 + y^2 + 2\cdot g\,x + 2\cdot f\,y + c = 0$

centre $= (-g, -f)$ radius $= r = \sqrt{(g^2 + f^2 - c)}$

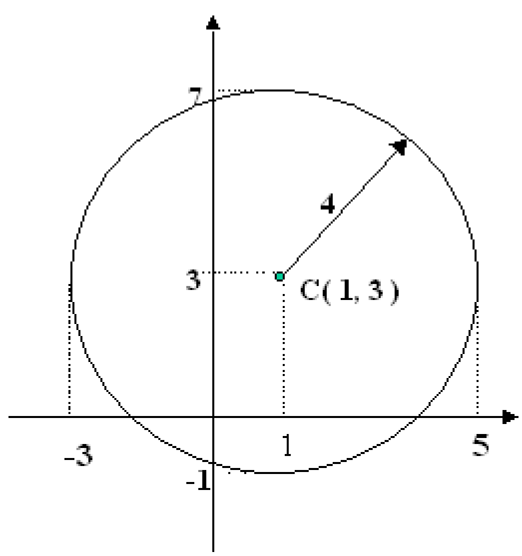
we have

$2x^2 + 2y^2 - 4x - 12y - 12 = 0$

$x^2 + y^2 - 2x - 6y - 6 = 0$

$2g = -2$ $2f = -6$

$g = -1$ $f = -3$



Hence we have

centre $= (-g, -f) = (1, 3)$

radius $= r = \sqrt{[(-1)^2 + (-3)^2 - (-6)]} = \sqrt{16} = 4$

Hence both equations represent the SAME circle

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2. The line with equation below cuts the circle in Q1 above.
Find the co-ordinates of intersection.

Line has equation
$x + y = 8$

Solution

Line has equation $x + y = 8$
Circle has equation $x^2 + y^2 - 2x - 6y - 6 = 0$

Rearrange line equation to $y = 8 - x$

Substitute for y in the circle equation and solve the quadratic. We have

$x^2 + (8 - x)^2 - 2x - 6(8 - x) - 6 = 0$

$x^2 + \left(x^2 - 16 \cdot x + 64\right) - 2x - 48 + 6x - 6 = 0$

$2 \cdot x^2 - 12x + 10 = 0$

$x^2 - 6x + 5 = (x - 5) \cdot (x - 1) = 0$

$x = 1$ And $x = 5$

Final sub the values obtained for x into the line equation to get values for y.

For

$x = 1 \quad y = 8 - 1 = 7$

Co-ordinates are $(1, 7)$

For
$x = 5 \quad y = 8 - 5 = 3$

Co-ordinates are $(5, 3)$

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3. Suppose you are given a line with a particular equation and you are told that this line does not touch or intersect with the circle in Q1.
Describe how you would prove this.

Solution

You would do the following:-

1. Put the line equation into the form
$$y = m \cdot x + c$$
2. Substitute for y into the circle equation

3. Put the resulting quadratic into the form
$$a \cdot x^2 + b \cdot x + c = 0$$

4. Final determine the value of
$$b^2 - 4 \cdot a \cdot c$$
$$b^2 - 4 \cdot a \cdot c > 0$$

2 real and distinct roots and hence intersects at 2 points

$b^2 - 4 \cdot a \cdot c = 0$ Equal roots

Hence touches at 1 points and is therefore a tangent

$b^2 - 4 \cdot a \cdot c < 0$ No real roots

Hence does not intersect or touch circle.

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4. For the circles given below determine whether or not they touch.

Circle 1: $\text{centre} = (-2, 0) \quad \text{radius} = 3$

Circle 2: $\text{centre} = (5, 0) \quad \text{radius} = 4$

Solution

If they touch then the following is true

Distance from C_1 to C_2 equals $r_1 + r_2$

We have distance from C_1 to C_2 $\sqrt{[[5-(-2)] + (0-0)]} = \sqrt{49} = 7$

and $r_1 + r_2 = 3 + 4 = 7$

Hence the circles do touch.

5. For the circle equation in Q1 determine the values that the constant can take to insure that the equation is the equation of a circle.

Solution

Circle equation

$x^2 + y^2 - 2x - 6y - 6 = 0 \qquad g = 1 \quad f = 3$

We need the radius to be greater than zero.

$\text{radius} = r = \sqrt{(g^2 + f^2 - c)} > 0$

$\sqrt{[(-1)^2 + (-3)^2 - c]} > 0$

Squaring both sides we get:-

$[(-1)^2 + (-3)^2 - c] > 0$

$(1 + 9 - c) > 0$

$10 - c > 0$

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$c < 10$

Hence to be the equation of a circle the constant term c must be less than 10.